Mass Spectra of Charmed and Bottomed Mesons in $1/m_Q$ Expansion

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Newly observed charmed and bottomed meson spectra are explained in our semi-relativistic quark potential model with $1/m_Q$ expansion. We discuss mass spectra of D, D_s, B and B_s mesons including their radial excitation.

§1. Introduction

In these years, new excited states of charmed and bottomed mesons were discovered by some collaborations. $D_0^*(2308)$ and $D_1'(2427)$ observed by Belle[1)] are identified as $c\bar{q}$ ($q = u/d$) excited states with the quantum numbers $J^P = 0^+$ and 1^+, respectively. Another set of heavy mesons, $D_{s0}(2317)$ by BaBar[2)] and $D_{s1}(2460)$ by CLEO[3)] are identified as $c\bar{s}$ excited states with the same quantum numbers as D. The decay widths of these excited D_{sJ} mesons are narrow since the masses are below DK/D^*K threshold and hence the dominant decay modes violate the isospin invariance, whereas those excited D mesons are broad because of no such restriction as in D_{sJ} cases.

After the discovery of these states, two new D_s, $D_{s0}(2860)$[5)] and $D_s^*(2715)$,[6)] were discovered in last year. Likewise some higher resonances of B/B_s particles were recently discovered. This situation opens a new era of spectroscopy of heavy-light mesons, which is challenging to theorists to solve these spectra.

However, a conventional potential model fails to explain their masses and decay widths. Other approaches have been already proposed to understand these new states. Some people claim that these states may be candidates of new exotic state like a Tetra-quark. Those approaches are not yet completely established even though they may be interesting themselves. In this paper, we give a new formalism to calculate the mass spectra of heavy-light mesons including their radial excitation with respect to the relativistic effect, and explain these new states as a conventional $Q\bar{q}$ meson.[8),9)]

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§2. Theoretical framework

Our model starts from an effective Hamiltonian for a two-body system of a heavy quark Q with mass m_Q and a light anti-quark $\bar{q}$ with mass $m_{\bar{q}}$,

$$H = (\vec{\alpha}_{\bar{q}} \cdot \vec{p}_{\bar{q}} + \beta_{\bar{q}} m_{\bar{q}}) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_{\bar{q}} \beta_Q S(r) + \left\{ 1 - \frac{1}{2} \left[\vec{\alpha}_{\bar{q}} \cdot \vec{\alpha}_Q + (\vec{\alpha}_{\bar{q}} \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n}) \right] \right\} V(r) \,, \tag{2·1}$$

where $\vec{\alpha}_i$ and β_i are gamma matrices, and $\vec{n} = \vec{r}/r$ is a unit vector along the relative coordinate between Q and $\bar{q}$. S and V are a scalar confining potential and a vector one-gluon exchange Coulomb potential with transverse interaction, which are given as

$$S(r) = \frac{r}{a^2} + b \,, \quad V(r) = -\frac{4}{3} \frac{\alpha_s}{r} \,, \tag{2·2}$$

where a and $1/b$ are parameters with length dimension and α_s is a strong coupling constant. The Hamiltonian representing kinetic terms of light and heavy quarks has Dirac structure, where the relativistic effect of quarks is respected. A wave function of heavy-light system has 16 components at this stage, because each wave function of heavy and light quarks has 4 components respectively. To solve an eigenvalue equation, we apply the Foldy-Wouthuysen-Tani (FWT) transformation to the heavy quark related operators in above Hamiltonian, which is equivalent to a nonrelativistic reduction of a heavy quark sector. It is reasonable because a heavy quark is sufficiently heavier than a light anti-quark. The eigenvalue equation for an atom-like bound state $Q\bar{q}$ can be given by

$$(H_{\rm FWT} - m_Q) \otimes \psi_{\rm FWT} = \tilde{E} \psi_{\rm FWT} \,, \tag{2·3}$$

where $\tilde{E} = E - m_Q$ (E being the bound state mass of $Q\bar{q}$) is the binding energy and a notation $\otimes$ denotes that gamma matrices of a light anti-quark is multiplied from left with the wave function while those of a heavy quark from right. The wave function $\psi_{\rm FWT}$ has 8 components, since a wave function of the heavy quark sector is reduced to 2 components by the FWT transformation. Then we expand the whole system, i.e., Hamiltonian, wave function, and eigenvalue is expanded in powers of $1/m_Q$ as

$$H_{\rm FWT} - m_Q = m_Q H_{-1} + H_0 + \frac{1}{m_Q} H_1 + \frac{1}{m_Q^2} H_2 + \cdots \,, \tag{2·4}$$

$$\tilde{E} = E - m_Q = E_0^a + \frac{1}{m_Q} E_1^a + \frac{1}{m_Q^2} E_2^a + \cdots \,, \tag{2·5}$$

$$\psi_{\rm FWT} = \psi_0^a + \frac{1}{m_Q} \psi_1^a + \frac{1}{m_Q^2} \psi_2^a + \cdots \,, \tag{2·6}$$

where a subscript i of H_i, E_i^a, and ψ_i^a stands for the i-th order in $1/m_Q$, and their explicit expression are presented in Refs. 7) and 8). The relativistic effect of a heavy quark is incorporated as higher order corrections of $1/m_Q$ into a $Q\bar{q}$ system.

§3. Numerical results and discussion

Using the perturbation method, the eigenvalue equation can be numerically solved in order by order in $1/m_Q$ expansion. We first determine the parameters by fitting the observed D and D_s meson masses with around one percent of accuracy compared with experiments. Subsequently, recently discovered $n = 2$ (the first radial excitation) masses are calculated at the same time. Here only a strong coupling constant α_s is modified and other parameters are kept the same as in Ref. 8), in which we have obtained $\alpha_s^{n=2} = 0.344$ both for D and D_s. These are presented in Table I at the first order of calculation in $1/m_Q$ expansion.

With these values of parameters, we obtain D and D_s meson masses as well as their radial excitations. The results of $n = 1$ states are shown in Tables II for D and IV for D_s mesons. We also predict $n = 2$ states, which are shown in Tables III and V with a different value of α_s, which may be actually different for B/B_s particles as well. The details of bottomed meson spectra are shown in Refs. 8) and 9). In these Tables, masses in the 0-th order calculation labeled by M_0 are degenerate for members of each spin doublet. c_i denotes the i-th order correction in $1/m_Q$ expansion (in this paper, we show only $i = 1$ results) and thus, the calculated heavy meson mass $M_{\rm calc}$ is given by the sum of M_0 and corrections up to the i-th order,

$$M_{\rm calc} = M_0 + \sum_{i=1}^{n} (p_i + n_i)\,, \tag{3·1}$$

where p_i and n_i are i-th order positive and negative component contributions of a heavy quark, respectively. Here one should notice that $M_0 = m_Q + E_0$. The degeneracy in the same spin doublets is automatically resolved by taking account of higher order corrections in $1/m_Q$ expansion.

As one can see in Tables II and IV, our calculated masses for $n = 1$ states are in good agreement with each observed value $M_{\rm obs}$, and the discrepancies are less than 1%. Especially it is remarkable that newly observed levels 0^+ and 1^+ of both D and D_s mesons are reproduced well, where the masses of D_{sJ} mesons are below DK/D^*K thresholds. These levels of D and D_s mesons cannot be reproduced by any other quark potential models which predict about $100 \sim 200$ MeV higher than our values. This result gives us a great confidence that our framework may give a good solution to the narrow D_{sJ} puzzle.

For the first radial excitation $n = 2$ states, the results are shown in Tables III and V where newly observed $D_{sJ}(2715)$ and $D_{sJ}(2856)$ mesons are assigned as 1^- and 0^+ states, respectively. Our results are consistent with experimental data, and we predict masses for other states. However, due to the lack of data, one may need further new data in order that our model give a conclusive explanation for the radial excitation.

Table I. Most optimal values of parameters.

Parameters	$\alpha_s^{n=1}$	$\alpha_s^{n=2}$	a (GeV^{-1})	b (GeV)	$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)
	0.261	0.344	1.939	0.0749	0.0112	0.0929	1.032

Table II. D meson mass spectra (units are in MeV).

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	$M_{\rm calc}$	$M_{\rm obs}$
$^1S_0(0^-)$	1784	0.476×10^{-1}	0.374×10^{-1}	1.013×10^{-2}	1869	1867
$^3S_1(1^-)$		1.271×10^{-1}	1.266×10^{-1}	0.512×10^{-3}	2011	2008
$^3P_0(0^+)$	2067	1.046×10^{-1}	0.959×10^{-1}	0.874×10^{-2}	2283	2308
"$^3P_1$"(1^+)		1.713×10^{-1}	1.689×10^{-1}	2.444×10^{-3}	2421	2427
"$^1P_1$"(1^+)	2125	1.415×10^{-1}	1.410×10^{-1}	0.486×10^{-3}	2425	2420
$^3P_2(2^+)$		1.618×10^{-1}	1.617×10^{-1}	1.364×10^{-4}	2468	2460

Table III. $D(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	$M_{\rm calc}$	$M_{\rm obs}$
$^1S_0(0^-)$	2241	1.078×10^{-1}	0.975×10^{-1}	1.038×10^{-2}	2483	–
$^3S_1(1^-)$		1.917×10^{-1}	1.910×10^{-1}	6.882×10^{-4}	2671	–
$^3P_0(0^+)$	2418	1.540×10^{-1}	1.444×10^{-1}	9.621×10^{-3}	2791	–
"$^3P_1$"(1^+)		2.493×10^{-1}	2.488×10^{-1}	5.352×10^{-4}	3021	–
"$^1P_1$"(1^+)	2491	2.076×10^{-1}	2.070×10^{-1}	5.956×10^{-4}	3008	–
$^3P_2(2^+)$		2.319×10^{-1}	2.318×10^{-1}	1.101×10^{-4}	3069	–

Table IV. D_s meson mass spectra (units are in MeV).

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	$M_{\rm calc}$	$M_{\rm obs}$
$^1S_0(0^-)$	1900	0.352×10^{-1}	0.270×10^{-1}	0.816×10^{-2}	1967	1969
$^3S_1(1^-)$		1.102×10^{-1}	1.098×10^{-1}	4.076×10^{-4}	2110	2112
$^3P_0(0^+)$	2095	1.101×10^{-1}	1.027×10^{-1}	0.740×10^{-2}	2325	2317
"$^3P_1$"(1^+)		1.779×10^{-1}	1.752×10^{-1}	2.620×10^{-3}	2467	2460
"$^1P_1$"(1^+)	2239	1.274×10^{-1}	1.270×10^{-1}	3.860×10^{-4}	2525	2535
$^3P_2(2^+)$		1.467×10^{-1}	1.466×10^{-1}	1.035×10^{-4}	2568	2572

Table V. $D_s(n=2)$ meson mass spectra (first order). Units are in MeV.

$^{2s+1}L_J(J^P)$	M_0	c_1/M_0	p_1/M_0	n_1/M_0	$M_{\rm calc}$	$M_{\rm obs}$
$^1S_0(0^-)$	2328	1.006×10^{-1}	0.919×10^{-1}	8.695×10^{-3}	2563	–
$^3S_1(1^-)$		1.830×10^{-1}	1.824×10^{-1}	5.744×10^{-4}	2755	2715
$^3P_0(0^+)$	2456	1.553×10^{-1}	1.470×10^{-1}	8.245×10^{-3}	2837	2856
"$^3P_1$"(1^+)		2.551×10^{-1}	2.543×10^{-1}	7.667×10^{-4}	3082	–
"$^1P_1$"(1^+)	2585	1.969×10^{-1}	1.966×10^{-1}	2.531×10^{-4}	3094	–
$^3P_2(2^+)$		2.209×10^{-1}	2.209×10^{-1}	5.070×10^{-7}	3157	–

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