CONSISTENT DEFINITIONS OF FLUX AND ELECTRIC AND MAGNETIC CURRENT IN ABELIAN PROJECTED $SU(2)$ LATTICE GAUGE THEORY

RICHARD W. HAYMAKER* AND TAKAYUKI MATSUKI†

Department of Physics and Astronomy, Louisiana State University,
Baton Rouge, Louisiana 70803-4001, USA
E-mail: haymaker@rouge.phys.lsu.edu,matsuki@tokyo-kasei.ac.jp

Through the use of a lattice $U(1)$ Ward-Takahashi identity, one can find a precise definition of flux and electric four-current that does not rely on the continuum limit. The magnetic four-current defined for example by the DeGrand-Toussaint construction introduces order a^2 errors in the field distributions. We advocate using a single definition of flux in order to be consistent with both the electric and magnetic Maxwell's equations at any lattice spacing. In a $U(1)$ theory the monopoles are slightly smeared by this choice, i.e. are no longer associated with a single lattice cube. In Abelian projected $SU(2)$ the consistent definition suggests further modifications. For simulations in the scaling window, we do not foresee large changes in the standard analysis of the dual Abrikosov vortex in the maximal Abelian gauge because the order a^2 corrections have small fluctuations and tend to cancel out. However in other gauges, the consequences of our definitions could lead to large effects which may help in understanding the choice of gauge. We also examine the effect of truncating all monopoles except for the dominant cluster on the profile of the dual Abrikosov vortex.

1. Introduction

Dual superconductivity has long been suggested as a possible mechanism for quark confinement signaled by a spontaneously broken $U(1)$ gauge symmetry and manifested by a dual Abrikosov vortex between quark and anti quark. This picture was verified some time ago in Abelian projected SU(2) lattice gauge theory in the maximal Abelian gauge[1]. More recently further studies have elaborated this picture[2,3,4].

*Talk presented at Confinement 2003, RIKEN, July 21 - 24, 2003 by R. Haymaker
†Permanent address: Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan

As in all lattice calculations, there is freedom in choosing lattice operators, requiring only that they agree with the continuum definition to lowest order in the lattice spacing a. However we have the opportunity in these studies to be more precise by incorporating the lattice Ward-Takahashi identity derived from the residual $U(1)$ gauge symmetry[5]. This gives an Ehrenfest relation for the expectation value of the fields and currents giving the electric Maxwell equations exactly at finite lattice spacing. Interestingly, this defines a unique lattice expression for the field strength or flux and the electric and magnetic currents.

In the present work, we examine the impact of this on the study of the dual Abrikosov vortex. To be consistent, the magnetic Maxwell equation must use the same definition as the electric one. However the standard DeGrand-Toussaint[6] [DT] definition of the magnetic current is based on a different definition of flux, resulting in inconsistencies in the magnetic Maxwell equation. We argue here that one should alter the DT construction, using a single definition of flux throughout. A consequence is that the magnetic current no longer contains discrete monopoles but rather a more general magnetic charge distribution. In effect, the monopoles are smeared in our picture.

This consistency question is only relevant at finite lattice spacing and all these concerns go away in the continuum limit. Nevertheless it is desirable to have a consistent treatment of total flux in the vortex determined from the electric Maxwell equation, and the profile shape determined by the magnetic Maxwell eqution at fixed lattice spacing. Further we note that the finite lattice spacing effects are significant for the values of β that we often use for calculations.

We also report on the effect of truncating DT monopole loops, keeping only the one large connected cluster. This truncation is expected to have no effect on the confinement signal[9,10]. This should manifest itself here in that the tail of the profile of the magnetic current of the vortex are unaffected by the truncation. We find this to be the case. This procedure requires that the magnetic current consists of discrete monopoles. Truncation is not well defined in the above smeared monopole picture. Hence we take the conventional view in presenting these results that the continuum limit is required to obtain the consistency described above.

2. Three definitions of flux

Let us consider three definitions of field strength, all agreeing to lowest order in a. The first definition was used by DT to define monopoles:

$$\begin{aligned}\widehat{F}^{(1)}_{\mu\nu} &= \theta_\mu(\boldsymbol{m}) - \theta_\mu(\boldsymbol{m}+\nu) - \theta_\nu(\boldsymbol{m}) + \theta_\nu(\boldsymbol{m}+\mu) - 2\pi n_{\mu\nu},\\ &\equiv \theta_{\mu\nu}(\boldsymbol{m}) - 2\pi n_{\mu\nu},\end{aligned}$$

where $\theta_\mu(\boldsymbol{m})$ refers to the $U(1)$ link angle in the domain $-\pi < \theta_\mu < +\pi$. The integers $n_{\mu\nu}$ are determined by requiring that $-\pi < \widehat{F}^{(1)}_{\mu\nu} < +\pi$. That is $\widehat{F}^{(1)}_{\mu\nu}$ is a periodic function of $\theta_{\mu\nu}$ with period 2π. Here quantities with $\widehat{}$ mean those which appear in the lattice numerical calculation without appending factors of e and a.

The second and third definitions gives the exact electric Maxwell equation for lattice averages

$$\Delta^-_\mu \langle \widehat{F}^{(i)}_{\mu\nu} \rangle_W = \langle \widehat{J}^{e(i)}_\mu \rangle_W, \qquad i = 2, 3, \tag{1}$$

where

$$\langle \cdots \rangle_W = \frac{\langle \cdots e^{i\theta_W} \rangle}{\langle e^{i\theta_W} \rangle},$$

for the cases of $U(1)$ gauge theory and $SU(2)$ gauge theory respectively. We give the $U(1)$ derivation in detail since it is straight forward and contains the essential points of the argument. There are significant complications in the $SU(2)$ case and hence we just sketch that derivation.

2.1. *Flux in the U(1) gauge theory*

Consider

$$Z_W(\epsilon_\mu(\boldsymbol{m})) = \int [d\theta] \sin\theta_W \exp\left(\beta S,\right)$$

$$S = \sum_{n,\ \mu>\nu} \cos\theta_{\mu\nu}(\boldsymbol{n}), \qquad \beta = \frac{1}{e^2}.$$

The subscript of $Z_W(\epsilon_\mu(\boldsymbol{m}))$ refers to the incorporation of the source into the partition function and the argument is a variable defined as the shift of one particular link, $\theta_\mu(\boldsymbol{m}) \to \theta_\mu(\boldsymbol{m}) + \epsilon_\mu(\boldsymbol{m})$. This translation can be transformed away since the measure is invariant under such an operation.

Therefore

$$0 = \delta Z_W = \int [d\theta] \sin\theta_W \exp(\beta S)\Big|_{\theta_\mu \to \theta_\mu + \epsilon_\mu} - \int [d\theta] \sin\theta_W \exp(\beta S)$$
$$= \epsilon_\mu \int [d\theta] \left(\delta_\mu(\boldsymbol{m}) \cos\theta_W + \sin\theta_W \frac{1}{e^2} \frac{\partial S}{\partial \theta_\mu} \right) \exp(\beta S) , \qquad (2)$$

where $\delta_\mu(\boldsymbol{m}) = \pm 1$ if $\boldsymbol{m}$ labels a $\pm$ directed link and $= 0$ otherwise. This is the static current generated by the Wilson loop.

$$\delta_\mu(\boldsymbol{m}) = \widehat{J}^e_\mu \qquad (3)$$

Next evaluate the derivative of S. Isolate the six plaquettes affected by the shift

$$S = \sum_{\nu \neq \mu} [\cos\{\theta_\mu(\boldsymbol{m}) + \theta_\nu(\boldsymbol{m}+\mu) - \theta_\mu(\boldsymbol{m}+\nu) - \theta_\nu(\boldsymbol{m})\}$$
$$+ \cos\{\theta_\mu(\boldsymbol{m}) - \theta_\nu(\boldsymbol{m}+\mu-\nu) - \theta_\mu(\boldsymbol{m}-\nu) + \theta_\nu(\boldsymbol{m}-\nu)\}] + \cdots ,$$

$$\frac{\partial S}{\partial \epsilon_\mu(\boldsymbol{m})} = \sum_{\nu \neq \mu} [-\sin\{\theta_\mu(\boldsymbol{m}) + \theta_\nu(\boldsymbol{m}+\mu) - \theta_\mu(\boldsymbol{m}+\nu) - \theta_\nu(\boldsymbol{m})\}$$
$$- \sin\{\theta_\mu(\boldsymbol{m}) - \theta_\nu(\boldsymbol{m}+\mu-\nu) - \theta_\mu(\boldsymbol{m}-\nu) + \theta_\nu(\boldsymbol{m}-\nu)\}] ,$$

$$\frac{\partial S}{\partial \epsilon_\mu(\boldsymbol{m})} = -\Delta^-_\nu \sin\theta_{\mu\nu}(\boldsymbol{m}). \qquad (4)$$

Using Eqn.(1) and Eqn.(4) we can see that Eqn.(2) is the form of Maxwell equations for averages.

$$\frac{1}{e^2} \Delta^-_\nu \langle \widehat{F}^{(2)}_{\mu\nu} \rangle_W = \widehat{J}^e_\mu(\boldsymbol{m}), \qquad (5)$$

where

$$\langle \cdots \rangle_W = \frac{\langle \sin\theta_W \cdots \rangle}{\langle \cos\theta_W \rangle}.$$

Since the charged line in a Wilson loop is closed the electric current is conserved. The local statement of conservation is

$$0 = \Delta^-_\mu \Delta^-_\nu \langle \widehat{F}^{(2)}_{\mu\nu} \rangle_W = \Delta^-_\mu \widehat{J}^e_\mu.$$

It is straightforward to verify on the lattice that the LHS of Eqn.(5) gives $-1, 0, 1$ depending on its position with respect to the Wilson loop. An alternative definition such as $\widehat{F}^{(1)}_{\mu\nu}$ need not vanish off the Wilson loop nor give ± 1 on the Wilson loop and hence would introduce an error. In a $U(1)$ theory there is no dynamical charge density, all charge resides on the Wilson loop.

2.2. *U(1) flux in the SU(2) theory in the maximal Abelian gauge*

We restrict our attention to the maximal Abelian gauge defined as a local maximum of

$$R = \sum_{n,\mu} \mathrm{tr}\left\{\sigma_3 U_\mu(n)\sigma_3 U_\mu^\dagger(n)\right\},$$

over the set of gauge transformations $\left\{g(m) = e^{i\alpha_i(m)\sigma_i}\right\}$, $U \longrightarrow U^g$. Taking U to be the stationary value, the stationary condition is given by

$$F_{jn}[U] = \left.\frac{\partial R[U^g]}{\partial \alpha_j(n)}\right|_{\alpha=0} = 0.$$

The second derivatives entering in the Jacobian are given by

$$M_{jn;im}(U) = \left.\frac{\partial^2 R[U^g]}{\partial \alpha_j(n)\partial \alpha_i(m)}\right|_{\alpha=0}.$$

The partition function is

$$Z_W^{g.f.}(\epsilon_\mu^3(\boldsymbol{m})) = \int [dU]\ \frac{1}{2}Tr[i\sigma_3 U_W(\boldsymbol{n})]\ \exp(\beta S)\prod_{jn}\delta(F_{jn}[U])\ \Delta_{FP}, \quad (6)$$

where the Faddeev-Popov Jacobian is

$$\Delta_{FP} = \det|M_{jn;im}(U)|.$$

An infinitesimal shift in this partition function has the added complication that it violates the gauge condition. This can be corrected by an infinitesimal accompanying gauge transformation. Thus the shift in one link affects all links. However experience has shown that the effect drops off rapidly with distance from the shifted link.

The derivative of the partition function Eqn.(6)with respect to $\epsilon_\mu^3(\boldsymbol{m})$ gives

$$\begin{aligned} 0 = &\int [dU]\left\{\delta_\mu(\boldsymbol{m})\frac{1}{2}Tr[U_W(\boldsymbol{n})]\right\}\exp(\beta S)\prod_{jn}\delta(F_{jn}[U])\Delta_{FP} \\ &+ \int [dU]\left\{\beta\frac{1}{2}Tr[i\sigma_3 U_W(\boldsymbol{n})]\frac{\partial S}{\partial \epsilon_\mu^3(\boldsymbol{m})}\right\}\exp(\beta S)\prod_{jn}\delta(F_{jn}[U])\Delta_{FP} \\ &+ \int [dU]\frac{1}{2}Tr[i\sigma_3 U_W(\boldsymbol{n})]\exp(\beta S)\frac{\partial}{\partial \epsilon_\mu^3(\boldsymbol{m})}\left\{\prod_{jn}\delta(F_{jn}[U])\Delta_{FP}\right\}. \quad (7) \end{aligned}$$

The third integral contains terms in the Ward Takahasni identity coming from the gauge fixing including ghost contributions.

We can cast this into the form of the electric Maxwell's equations for averages as in the case of the U(1) theory. However there are now more terms in the current. Consider the standard U(1) parametrization of an SU(2) element:

$$\begin{aligned} U_\mu(\boldsymbol{n}) &= \begin{pmatrix} C_\mu(\boldsymbol{n})e^{i\theta_\mu(\boldsymbol{n})} & S_\mu(\boldsymbol{n})e^{i\{\gamma_\mu(\boldsymbol{n})-\theta_\mu(\boldsymbol{n})\}} \\ -S_\mu(\boldsymbol{n})e^{-i\{\gamma_\mu(\boldsymbol{n})-\theta_\mu(\boldsymbol{n})\}} & C_\mu(\boldsymbol{n})e^{-i\theta_\mu(\boldsymbol{n})} \end{pmatrix}, \\ &= \begin{pmatrix} C_\mu & S_\mu e^{i\gamma_\mu} \\ -S_\mu e^{-i\gamma_\mu} & C_\mu \end{pmatrix} \begin{pmatrix} e^{i\theta_\mu} & 0 \\ 0 & e^{-i\theta_\mu} \end{pmatrix}, \end{aligned}$$

where

$$\begin{aligned} C_\mu(\boldsymbol{n}) &\equiv \cos\phi_\mu(\boldsymbol{n}), \\ S_\mu(\boldsymbol{n}) &\equiv \sin\phi_\mu(\boldsymbol{n}). \end{aligned} \tag{8}$$

In the Abelian projection factored form, the righthand factor contains the $U(1)$ photon, parameterized by θ. The lefthand factor contains the charged coset matter fields, parameterized by ϕ and γ. The transformation properties are well known and reviewed in DiCecio et.al.[5].

We consider an alternative separation into diagonal and off-diagonal parts which is needed in defining the flux.

$$\begin{aligned} U_\mu(\boldsymbol{n}) &= \begin{pmatrix} C_\mu e^{i\theta_\mu} & 0 \\ 0 & C_\mu e^{-i\theta_\mu} \end{pmatrix} + \begin{pmatrix} 0 & S_\mu e^{i(\gamma_\mu-\theta_\mu)} \\ -S_\mu e^{-i(\gamma_\mu-\theta_\mu)} & 0 \end{pmatrix}, \\ &= D_\mu(\boldsymbol{n}) + O_\mu(\boldsymbol{n}). \end{aligned}$$

The off-diagonal part is the charged matter field $\Phi_\mu \equiv S_\mu e^{-i(\gamma_\mu-\theta_\mu)}$. and diagonal part includes the photon, $e^{i\theta_\mu}$, but also a neutral remnant of the matter field $\sqrt{1-|\Phi_\mu|^2}$ which $\rightarrow 1$ in the limit $a \rightarrow 0$.

To cast Eqn.(7) into the form of a current conservation law, we first consider the terms to zeroth order in O_μ. First the Wilson loop. Isolating the diagonal contributions gives

$$U_W = D_W + \widetilde{U}_W,$$

where D_W is the product of the diagonal parts

$$\begin{aligned} \frac{1}{2}Tr[D_W(\boldsymbol{n})] &= \prod_W C_\mu(\boldsymbol{n}) \cdot \boldsymbol{\cos\theta_W}, \\ \frac{1}{2}Tr[i\sigma_3 D_W(\boldsymbol{n})] &= -\prod_W C_\mu(\boldsymbol{n}) \cdot \boldsymbol{\sin\theta_W}. \end{aligned}$$

We adhere to the standard choice of an Abelian Wilson loop in which we drop any contributions due to off diagonal elements O_μ and further take the factors $C_\mu(\boldsymbol{n}) = 1$ giving

$$\frac{1}{2}Tr[U_W^{\rm Abelian}(\boldsymbol{n})] = \cos\theta_W,$$
$$\frac{1}{2}Tr[i\sigma_3 U_W^{\rm Abelian}(\boldsymbol{n})] = -\sin\theta_W.$$

Second, consider the action. Write

$$S = \sum_{n,\ \mu>\nu} \frac{1}{2}{\rm Tr}[D_{\mu\nu}(\boldsymbol{n})] + \widetilde{S},$$

where $\widetilde{S}$ contains terms involving $O_\mu(\boldsymbol{n})$.

$$\frac{\partial(S-\widetilde{S})}{\partial\epsilon_\mu^3(\boldsymbol{m})} = \sum_{\nu\neq\mu}\Big[\frac{1}{2}{\rm Tr}\left\{-i\sigma_3(\boldsymbol{m})D_\mu(\boldsymbol{m})D_\nu(\boldsymbol{m}+\mu)D_\mu^\dagger(\boldsymbol{m}+\nu)D_\nu^\dagger(\boldsymbol{m})\right\}$$
$$+\frac{1}{2}{\rm Tr}\left\{-i\sigma_3(\boldsymbol{m})D_\mu(\boldsymbol{m})D_\nu^\dagger(\boldsymbol{m}+\mu-\nu)D_\mu^\dagger(\boldsymbol{m}-\nu)D_\nu(\boldsymbol{m}-\nu)\right\}\Big].$$

Since all matrices are diagonal we can simplify:

$$\frac{\partial(S-\widetilde{S})}{\partial\epsilon_\mu^3(\boldsymbol{m})} = \sum_{\nu\neq\mu}\Delta_\nu^-\left[C_\mu(\boldsymbol{m})C_\nu(\boldsymbol{m}+\mu)C_\mu(\boldsymbol{m}+\nu)C_\nu(\boldsymbol{m})\sin\theta_{\mu\nu}(\boldsymbol{m})\right].$$

The quantity in square brackets is antisymmetric in $\mu\nu$ and we identify this as proportional to the field tensor.

$$\widehat{F}_{\mu\nu}^{(3)} \equiv C_\mu(\boldsymbol{m})C_\nu(\boldsymbol{m}+\mu)C_\mu(\boldsymbol{m}+\nu)C_\nu(\boldsymbol{m})\sin\theta_{\mu\nu}(\boldsymbol{m})$$

Returning to the identity, Eqn.(7), and using the notation

$$\int[dU]\{\cdots\}\exp(\beta S)\prod_{jn}\delta(F_{jn}[U])\Delta_{FP} = \langle\cdots\rangle_{g.f.}$$

we obtain

$$0 = \delta_\mu(\boldsymbol{m})\langle\cos\theta_W\rangle_{g.f.} - \beta\langle\sin\theta_W\Delta_\nu^-\widehat{F}_{\mu\nu}^{(3)}\rangle_{g.f.} - \beta\langle\sin\theta_W\frac{\partial\widetilde{S}}{\partial\epsilon_\mu^3(\boldsymbol{m})}\rangle_{g.f.}$$
$$+ \text{gauge fixing terms} + \text{ghost terms}\,.$$

Rearranging terms as in the $U(1)$ case, we get

$$\beta\frac{\langle\sin\theta_W\Delta_\nu^-\widehat{F}_{\mu\nu}^{(3)}\rangle_{g.f.}}{\langle\cos\theta_W\rangle_{g.f.}} = \delta_\mu(\boldsymbol{m}) - \beta\frac{\langle\sin\theta_W\frac{\partial\widetilde{S}}{\partial\epsilon_\mu^3(\boldsymbol{m})}\rangle_{g.f.}}{\langle\cos\theta_W\rangle_{g.f.}} + \frac{\text{g.f. \& ghosts}}{\langle\cos\theta_W\rangle_{g.f.}},$$

$$\Delta_\nu^- \langle F^{(3)}_{\mu\nu} \rangle_{W,g.f.} = \widehat{J}_\mu^{(\mathrm{e})\ \text{Abelian Wilson loop}} + \widehat{J}_\mu^{(\mathrm{e})\ \text{matter fields}} + \widehat{J}_\mu^{(\mathrm{e})\ \text{gauge fixing}} + \widehat{J}_\mu^{(\mathrm{e})\ \text{ghosts}}. \tag{9}$$

The 'Abelian Wilson loop' term is analogous to the above $U(1)$ case. The 'charged matter field' term arises from the off-diagonal elements of links in the action expression and would contribute without gauge fixing. The 'gauge fixing' term arises from the corrective gauge tranformation that accompanies the shift of a link. The 'ghost' term arises from the shift and accompanying gauge transformation on the Faddeev-Popov determinant. See DiCecio et.al.[5] for a complete derivaton of all terms.

It is important here to note that we are not interested in distinguishing the various contributions to the current in the present work. We are only interested in the total current and that can be obtained from the LHS of Eqn.(9).

3. Consistency with the magnetic Maxwell equation

Having defined a unique flux $\widehat{F}^{(i)}_{\mu\nu}$ through the electric Maxwell equation, the magnetic Maxwell equation is

$$\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu^+ \widehat{F}^{(i)}_{\rho\sigma} = \widehat{J}^{m(i)}_\mu \qquad i = 2,3.$$

However the standard DT definition of current is

$$\widehat{J}^{m(1)}_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu^+ \widehat{F}^{(1)}_{\rho\sigma}.$$

and hence if we use the conventional $\widehat{F}^{(1)}$ to define the monopole current, and $\widehat{F}^{(2)}$ or $\widehat{F}^{(3)}$ respectively for $U(1)$ and $SU(2)$ theories to get an exact expression for flux in the confining string, then the magnetic Maxwell equation is violated.

The solution is to relax the requirement that we use the DT monopole definition and use $\widehat{F}^{(2)}$ or $\widehat{F}^{(3)}$ instead when defining monopoles. A simple configuration for the $U(1)$ case ($\widehat{F}^{(2)}$) will illustrate the effect. Consider a single DT monopole with equal flux out of the six faces of the cube. Then the ratio of the $\widehat{F}^{(2)}$ flux out of this cube compared to the DT $\widehat{F}^{(1)}$ flux gives

$$\frac{6\sin(2\pi/6)}{6(2\pi/6)} \approx 0.83.$$

Since the current is conserved, the balance is made up by magnetic charge in the neighboring cells. On a large surface the total flux is the same for the two definitions.

In Fig.(1) we plot the $\widehat{F}^{(1)}_{\mu\nu}$ as a function of $\theta_{\mu\nu}$, giving a "sawtooth" shape. Monopoles occur as a consequence of $\widehat{F}^{(1)}$ crossing the sawtooth edge, giving a mismatch of 2π in the flux out of a cube. The sine function has no such discreteness and so the notion of discrete Dirac strings and Dirac monopoles is modified. However as one approaches the continuum limit, the action will drive the plaquette to zero and then the regions where the sawtooth differs from the sine are suppressed. Hence we expect both forms to give the standard Dirac picture in the continuum limit.

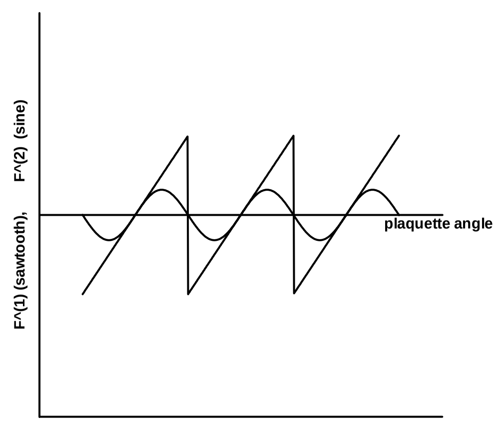

Figure 1. $\widehat{F}^{(1)}_{\mu\nu}$ (sawtooth) and $\widehat{F}^{(2)}_{\mu\nu}$ (sine) as a function of the plaquette angle $\theta_{\mu\nu}$

The electric Maxwell equation determines the total flux in the confining string and the magnetic Maxwell equation determines the transverse shape. Further the latter enters directly in the determination of the London penetration length, λ_d. To see this let us consider the classical Higgs theory which we use to model the simulation data. The dual field is given by

$$\widehat{G}_{\mu\nu}(\boldsymbol{m}) = \Delta^-_\mu \theta^{(d)}_\nu(\boldsymbol{m}) - \Delta^-_\nu \theta^{(d)}_\mu(\boldsymbol{m}),$$

where $\theta^{(d)}_\mu(\boldsymbol{m})$ is a dual link variable. Let us choose to break the gauge symmetry spontaneously through a constrained Higgs field.

$$\Phi(\boldsymbol{m}) = v\rho(\boldsymbol{m}) e^{i\chi(\boldsymbol{m})}, \qquad \rho(\boldsymbol{m}) = 1.$$

Under these conditions the magnetic current simplifies to

$$\widehat{J}^m_\mu(\boldsymbol{m}) = 2e_m^2 v^2 \sin\left\{\theta_\mu(\boldsymbol{m}) + \chi_\mu(\boldsymbol{m}+\mu) - \chi_\mu(\boldsymbol{m})\right\}.$$

where e_m is the magnetic coupling. The magnetic Maxwell equation is

$$\Delta^+_\mu \widehat{G}_{\mu\nu} = \widehat{J}^m_\nu. \tag{10}$$

For small $\theta^{(d)}$ it is easy to see that there is a London relation of the form

$$\widehat{G}_{\mu\nu}(\boldsymbol{m}) = \frac{1}{2e_m^2 v^2}\left(\Delta^-_\mu \widehat{J}^m_\nu(\boldsymbol{m}) - \Delta^-_\nu \widehat{J}^m_\mu(\boldsymbol{m})\right). \tag{11}$$

Taking the confining string along the 3 axis and choosing $\mu = 1$ and $\nu = 2$ we see that the profile of the third component of curl of the magnetic current must match the third component of the electric flux profile. This assumes an infinite Higgs mass M_H. With a finite mass there is a transition region of size $\sim 1/M_H$ in the core of the vortex but the above London relation must hold sufficiently far outside the core.

Combining Eqns.(10) and (11) we get the relation

$$\left(1 - \lambda_d^2 \Delta^+_\mu \Delta^-_\mu\right)\left\langle \widehat{E}_3(\boldsymbol{m})\right\rangle_W = 0, \lambda_d^{-1} = e_m v\sqrt{2}$$

The corresponding equations in the simulation of the $SU(2)$ theory must also be satisfied in order to arrive at this correct expression for λ_d and hence the importance of our definitions.

4. Numerical Results

We generated 208 configurations on a 32^4 lattice at $\beta = 2.5115$. The maximal Abelian gauge fixing used over-relaxation. Fig.(2) shows the profile of the electric flux corresponding to $\widehat{F}^{(2)}$ and $\widehat{F}^{(3)}$. Fig.(3) shows the profile of the theta component of the magnetic current corresponding to $\widehat{F}^{(1)}$, $\widehat{F}^{(2)}$ and $\widehat{F}^{(3)}$.

In summary, we showed that consistency requires one use the same definition of flux throughout. If, for example, one uses $\widehat{F}^{(3)}$ definition of electric field (bottom graph in Fig.(2)) in order to account correctly for the total flux but then uses the DT definition of current (top graph in Fig.(3)) we would then incur errors of $\sim 40\%$. For simulations in the scaling window, as long as we use the same definition consistently we do not foresee large changes in the standard analysis of the dual Abrikosov vortex in the maximal Abelian gauge because the order a^2 corrections have small fluctuations and tend to cancel out[8]. However in other gauges, the consequences of our definitions could lead to large effects which may help in understanding the choice of gauge.

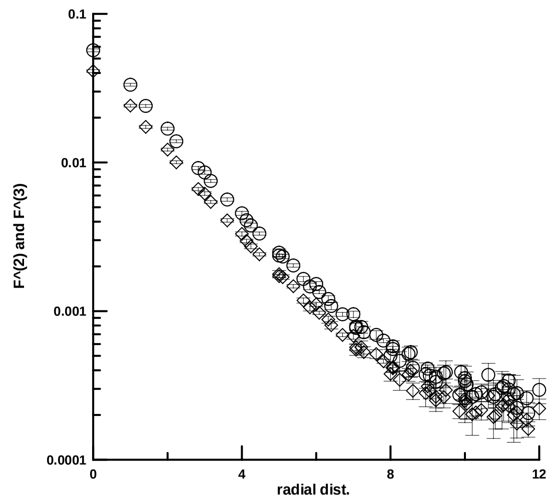

Figure 2. Profile of the electric field (highest to lowest) $\widehat{E}_3^{(2)}$ (circles) and $\widehat{E}_3^{(3)}$ (diamonds) on the mid-plane between q and $\bar{q}$ separated by $13a$.

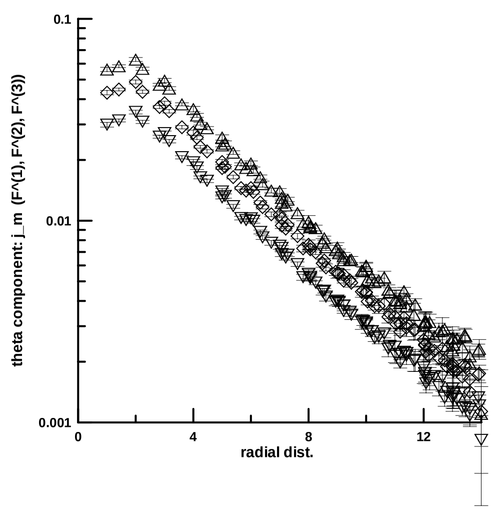

Figure 3. Profile of the theta component of the magnetic current on the mid-plane between q and $\bar{q}$ separated by $13a$ based on (highest to lowest) $\widehat{F}_{\mu\nu}^{(1)}$ (triangles), $\widehat{F}_{\mu\nu}^{(2)}$ (diamonds) and $\widehat{F}_{\mu\nu}^{(3)}$ (inverted triangles).

Finally we report on the effect of truncating DT monopole loops, keeping only the one large connected cluster[9,10]. As mentioned in the Introduction this truncation is expected to have no effect on the confinement signal. This should manifest itself here in that the tail of the profile of the electric field and magnetic current of the vortex are unaffected by the truncation. This is born out as expected. Fig.(4) shows that radial profile of the magnetic current is indistiguishable except for a small deviation in the core of the vortex.

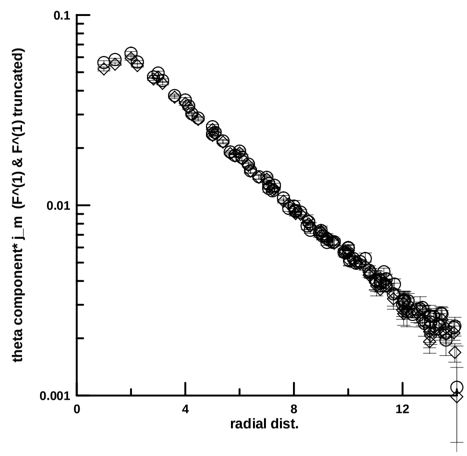

Figure 4. Profile of the theta component of the magnetic current on the mid-plane between q and $\bar{q}$ separated by $13a$ based on (highest to lowest) $\widehat{F}^{(1)}_{\mu\nu}$ (circles), $\widehat{F}^{(1)}_{\mu\nu}$ truncated (diamonds).

Acknowledgments

This work is supported in part by the U. S. Department of Energy under grant no. De-FG05-01 ER 40617.

References

1. V. Singh, D. A. Browne and R. W. Haymaker, *Phys. Lett.* **B306** (1993) 115.
2. G. S. Bali, C. Schlicher and K. Schilling, *Prog. Theor. Phys. Suppl.* **131** (1998) 645.
3. F. V. Gubarev, E.-M Ilgenfritz, M. I. Polikarpov, and T. Suzuki, *Phys. Lett.* **B468** (1999) 134.
4. Y. Koma, M. Koma, E.-M. Ilgenfritz, T. Suzuki, and M.I. Polikarpov, hep-lat/0302006.
5. G. DiCecio, A. Hart and R. Haymaker, *Phys. Lett.* **B441** (1998) 319.
6. T. A. DeGrand and D. Toussaint, *Phys. Rev.* **D22** (1980) 2478.
7. M. Zach, M. Faber, W. Kainz and P. Skala *Phys. Lett.* **B358** (1995) 325.
8. G. Poulis, *Phys. Rev.* **D54** (1996) 6974.
9. A. Hart and M. Teper, *Phys. Rev.* **D58** (1998) 014504.
10. F.V.Gubarev, A.V.Kovalenko, M.I.Polikarpov, S.N.Syritsyn, V.I.Zakharov, hep-lat/0212003; V.G. Bornyakov, P.Yu. Boyko, M.I. Polikarpov, V.I. Zakharov,hep-lat/0305021.