

# Heavy Meson Spectroscopy

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**RCNP研究会**

**『Challenge to New Exotic Hadrons with Heavy Quarks』**

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- **Potential Model Approach within  $1/m_Q$  Expansion**
  - Effective Hamiltonian and Eigenvalue Equation
  - Numerical Results
  - Radial Excitation
- **Summary**



# New Charm Spectra

TABLE I: New Heavy Mesons

state	mass (MeV)	width (MeV)	production/decay mode	comments	ref
$h_c$	$3524.4 \pm 0.6 \pm 0.4$	–	$\psi(2S) \rightarrow \pi^0 h_c \rightarrow (\gamma\gamma)(\gamma\eta_c)$	$\approx$ CQM / tests spin dependence	CLEO[83]
$\eta'_c$	$3654 \pm 6 \pm 8$	$< 55$	$B \rightarrow K\eta'_c \rightarrow KK_S K\pi$	$\approx$ CQM / tests hyperfine splitting	Belle[86]
	$3642.9 \pm 3.1 \pm 1.5$	$6.3 \pm 12.4 \pm 4.0$	$e^+e^- \rightarrow \eta'_c J/\psi$		CLEO[88]
	$3630.8 \pm 3.4 \pm 1.0$	$17.0 \pm 8.0 \pm 2.5$	$\gamma\gamma \rightarrow \eta'_c \rightarrow K_S K\pi$		BaBar[89]
$X(3872)$	$3872.0 \pm 0.6 \pm 0.5$	$< 2.3$ 95% C.L.	$B \rightarrow KX \rightarrow K\pi\pi J/\psi$	molecule, cusp, tetraquark	Belle[31]
	$3873.4 \pm 1.4$	–	$B \rightarrow KX \rightarrow K\pi\pi J/\psi$		BaBar[34]
	–	–	$B \rightarrow X \rightarrow \pi\pi\pi J/\psi$		Belle[42]
	–	–	$B \rightarrow X \rightarrow \gamma J/\psi$		Belle[42]
	$3871.3 \pm 0.7 \pm 0.4$	–	$p\bar{p} \rightarrow X \rightarrow \pi\pi J/\psi$		CDF[32]
	$3871.8 \pm 3.1 \pm 3.0$	–	$p\bar{p} \rightarrow X \rightarrow \pi\pi J/\psi$		DØ[33]
	avg = $3871.9 \pm 0.5$				
$X(3940)$	$3943 \pm 6 \pm 6$	$< 52$	$e^+e^- \rightarrow J/\psi X \rightarrow J/\psi DD^*$	$\chi'_{c1}, \eta'_c$ / needs confirmation	Belle[91]
$Y(3940)$	$3943 \pm 11 \pm 13$	$87 \pm 22 \pm 26$	$B \rightarrow KY \rightarrow K\pi\pi\pi J/\psi$	needs confirmation	Belle[99]
$Z(3930)$	$3931 \pm 4 \pm 2$	$20 \pm 8 \pm 3$	$\gamma\gamma \rightarrow Z \rightarrow DD$	$\chi'_{c2}$ / $\approx$ CQM	Belle[101]
$Y(4260)$	$4259 \pm 8 \pm 4$	$88 \pm 23 \pm 5$	$e^+e^- \rightarrow \gamma_{ISR} Y \rightarrow \gamma_{ISR} J/\psi\pi\pi$	hybrid? / needs confirmation	BaBar[103]
$D_s(2317)$	$2317.3 \pm 0.4 \pm 0.8$	$< 10$	$e^+e^- \rightarrow D_s(2317) \rightarrow D_s\pi^0$	molecule, tetraquark, shifted $c\bar{s}$	BaBar[114]
	$2319.8 \pm 2.1 \pm 2.0$	$\approx 0$	$B \rightarrow DD_s(2317) \rightarrow DD_s\pi^0$		Belle[130]
	$2318.5 \pm 1.2 \pm 1.1$		$D_s(2317) \rightarrow D_s\pi^0$		CLEO[123]
$D_s(2460)$	$2463.6 \pm 1.7 \pm 1.2$	$< 7$ 90% C.L.	$D_s(2460) \rightarrow D_s^*\pi^0$	molecule, tetraquark, shifted $c\bar{s}$	CLEO[123]
	$2458.0 \pm 1.0 \pm 1.0$	resolution	$D_s(2460) \rightarrow D_s\pi^0\gamma$		BaBar[125]
	$2459.2 \pm 1.6 \pm 2.0$	$\approx 0$	$B \rightarrow DD_s(2460) \rightarrow DD_s^*\pi^0, DD_s\gamma$		Belle[130]
$D_s(2630)$	$2632.6 \pm 1.6$	$< 17$ 90% C.L.	$D_s \rightarrow D^0 K^+$ and $D_s\eta$	artefact	SELEX[154]
$B_c$	$6285.7 \pm 5.3 \pm 1.2$	$0.474 \pm 0.07 \pm 0.33$ ps	$p\bar{p} \rightarrow B_c \rightarrow J/\psi\pi^\pm$	$\approx$ CQM	CDF[166]



# Open Charm Events

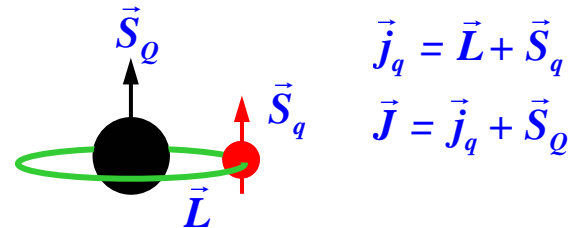
$D_0^*(2308)$  ,  $D_1'(2427)$

$D_{s0}^*(2317)$  ,  $D_{s1}(2460)$

$D_{sJ}(2715)$  ,  $D_{sJ}(2860)$

# Heavy-light Meson

- Heavy-light System:  $Q\bar{q}$   
→ “QCD hydrogen”



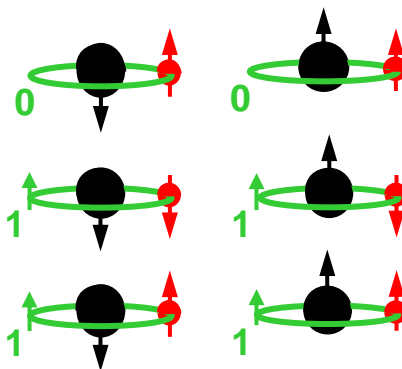
- The system is classified by  $j_q$ , total angular momentum of light quark degrees of freedom in the heavy quark limit.

- (Heavy) Spin Doublet

$$L=0, j_q = 1/2 \quad J^P = (0^-, 1^-)$$

$$L=1, j_q = 1/2 \quad J^P = (0^+, 1^+)$$

$$L=1, j_q = 3/2 \quad J^P = (1^+, 2^+)$$



Degenerate  
in the heavy  
quark limit

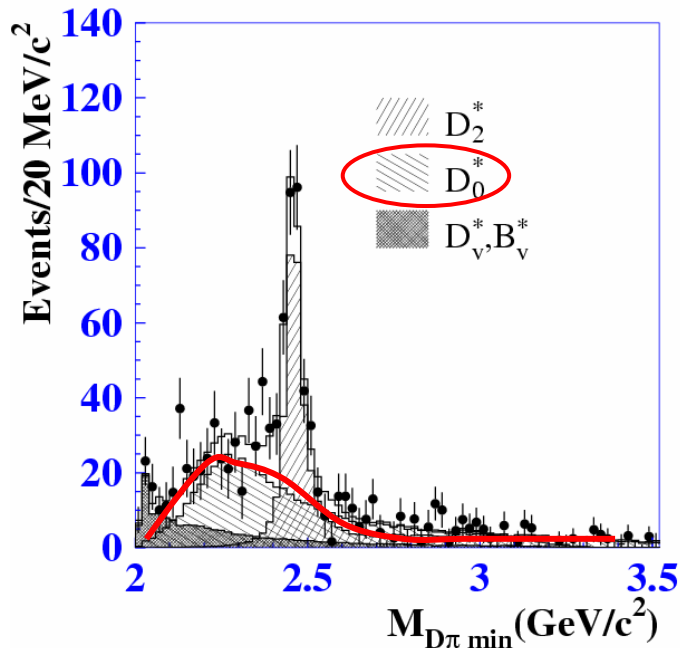
- $(0^-, 1^-)$  and  $(1^+, 2^+)$  doublets agree with theoretical calculation.
- **But, there are discrepancies for  $(0^+, 1^+)$  states.**



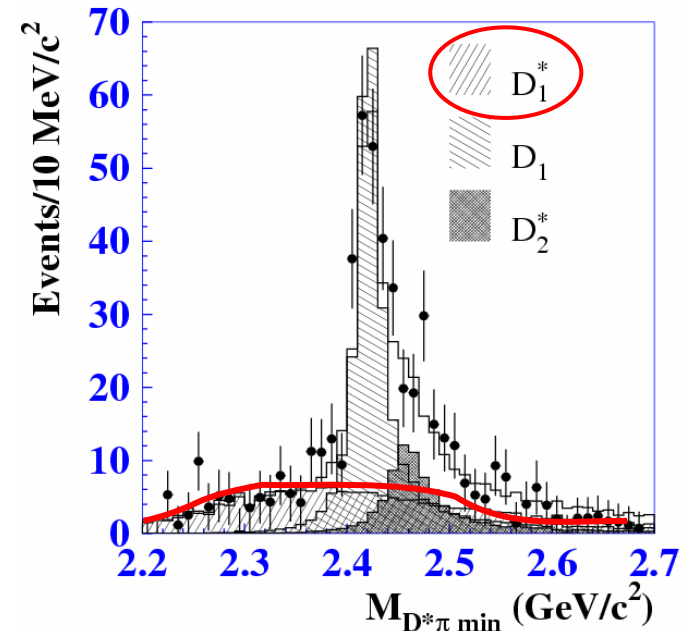
# Non-Strange ( $0^+, 1^+$ ) Doublet

- $D_0^*(2308)[0^+]$  and  $D_1'(2427)[1^+]$ 
  - Belle, PRL91, 262002 (2003); PRD69, 112002 (2004)
- Very broad widths: through  $\pi$  s-wave decay

$$B^- \rightarrow D^{**0} \pi^-, D^{**0} \rightarrow D^+ \pi^-$$



$$B^- \rightarrow D^{**0} \pi^-, D^{**0} \rightarrow D^{*+} \pi^-$$



# Strange ( $0^+, 1^+$ ) Doublet

- $D_{s0}^*(2317)[0^+]$  and  $D_{s1}(2460)[1^+]$ 
  - BaBar, PRL90, 242001 (2003); CLEO, PRD68, 032002 (2003)
- **Very narrow widths: below  $DK/D^*K$  thresholds**

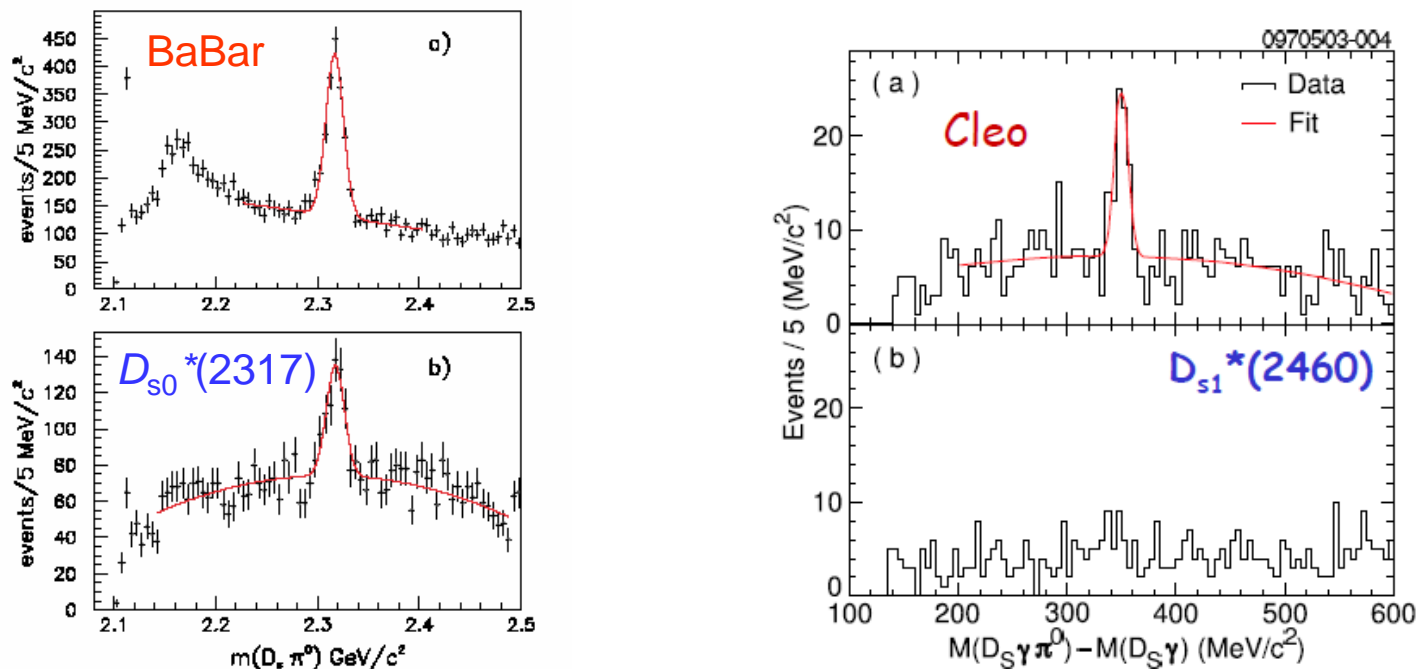
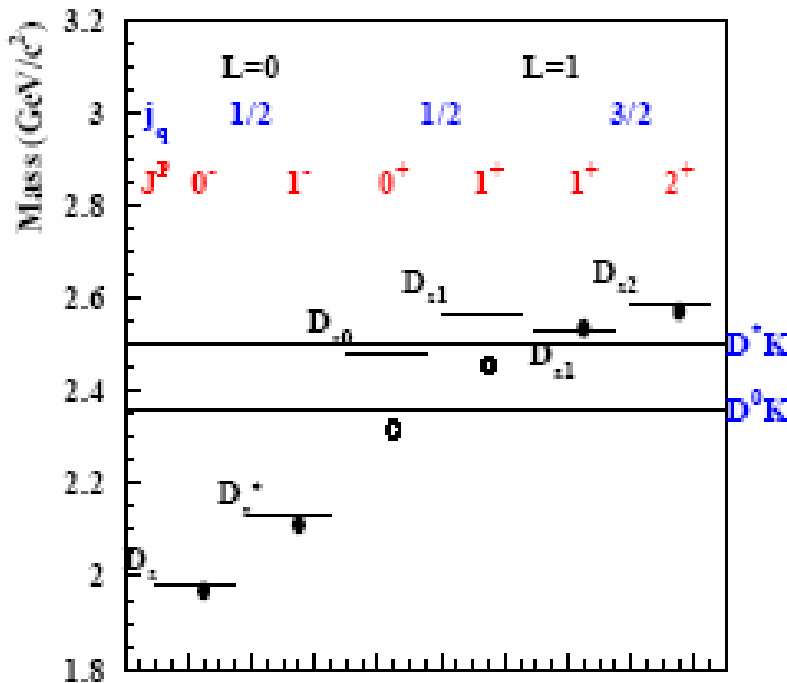


FIG. 2 (color online). The  $D_s^+ \pi^0$  mass distribution for (a) the decay  $D_s^+ \rightarrow K^+ K^- \pi^+$  and (b) the decay  $D_s^+ \rightarrow K^+ K^- \pi^+ \pi^0$ . The fits to the mass distributions as described in the text are indicated by the curves.



# $D_{sJ}$ Puzzle



K. Abe, Talk at PENTAQUARK04,  
July 20-23, 2004.

- already observed
- newly observed
- prediction by conventional potential model  
(Godfrey et al., PRD43, 1679 (1991))

- **Discrepancies with predictions** for  $L=1$  ( $0^+$  and  $1^+$ ) states

- Masses are smaller ( $\sim 160$  MeV)
- Decay width are narrow
- Below  $DK$  ( $D^*K$ ) threshold



Something odd must happen!?

- Isospin violating decay mode:  $D_{sJ}(0^+, 1^+) \rightarrow D_{sJ}(0^-, 1^-) + \pi$



# Exotic States?

- **Tetraquark**

- K. Terasaki, PRD68, 011501 (2003)

- Explain the decay property of  $D_s(0^+)$  as a Tetraquark

- V. Dmitrasinovic, PRL94, 162002 (2005)

- $D(0^+)$  and  $D_s(0^+)$  are in the anti-symmetric  $3_A^*$  multiplet

$$|D^0 \in \bar{3}_A\rangle = \frac{1}{2} |c(s(\bar{u}\bar{s} - \bar{s}\bar{u}) - d(\bar{d}\bar{u} - \bar{u}\bar{d}))\rangle$$



**Same Mass!**

$$|D_s \in \bar{3}_A\rangle = \frac{1}{2} |c(u(\bar{u}\bar{s} - \bar{s}\bar{u}) - d(\bar{d}\bar{s} - \bar{s}\bar{d}))\rangle$$

- Always contain color-singlet\*singlet component → broad width

- Where are those partner states in same multiplet?

- **Coupled Channel / Meson Molecule** (T. Barnes et al., PRD68, 054006 (2003))

- S-wave  $DK$  ( $D^*K$ ) continuum

- Very close to  $DK$  ( $D^*K$ ) thresholds (46 MeV)

- Where are ordinary ( $0^+$ ,  $1^+$ ) states in QM?



# Conventional $c\bar{s}$ ?

- **Chiral Effective Theory**

- **Ginzburg-Landau Type** (Bardeen et al., PRD68, 054024 (2003))

- **Vector Manifestation based on Hidden Local Symmetry**

(M. Harada, M. Rho, C. Sasaki, PRD70, 074002 (2004).)

Above approaches can reproduce **only mass differences** between parity doublets ( $0^-$ ,  $0^+$ ) and ( $1^-$ ,  $1^+$ ), not absolute values.

- It works in  $D_s$  meson, but it seems to not work in  $D$  meson system.

- **Potential Model**

- **Relativized model** (Godfrey et al., PRD43, 1697 (1991))

- $1/m_Q$  expansion (Our model, PTP117, 1077 (2007); PLB606, 329 (2005))

- **QCD Sum Rule**

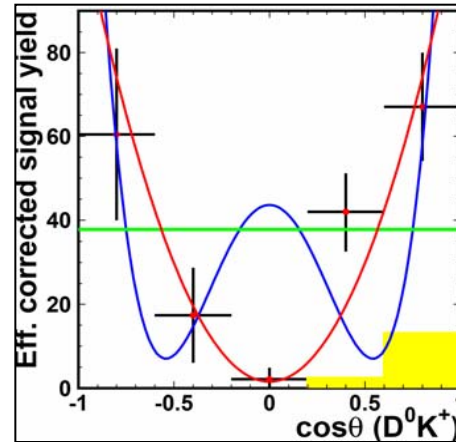
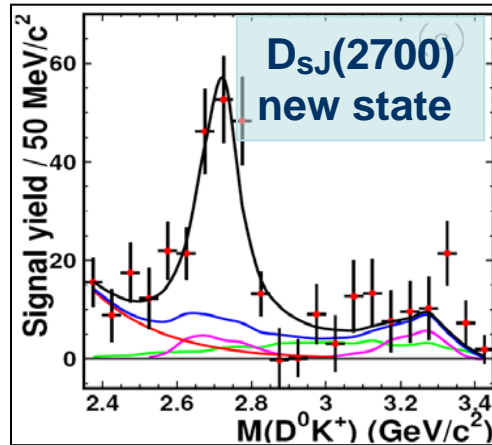
- **Branching ration is consistent with data** (Wei, Zhu, PRD06; Lu, Zhu, PRD06; Colangelo, PRD05)

- **Lattice**

- **Masses are still larger than experimental data**

# Other New $D_{sJ}$ States

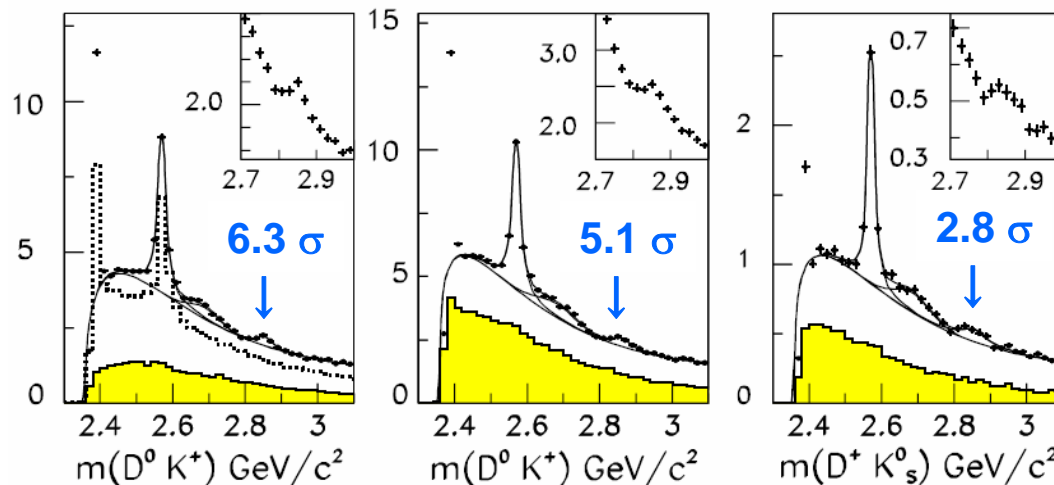
- $D_{sJ}(2715)$  (Belle, hep-ex/0608031)



J=0  
J=1  
J=2

$J^P = 1^-$   
2S state?

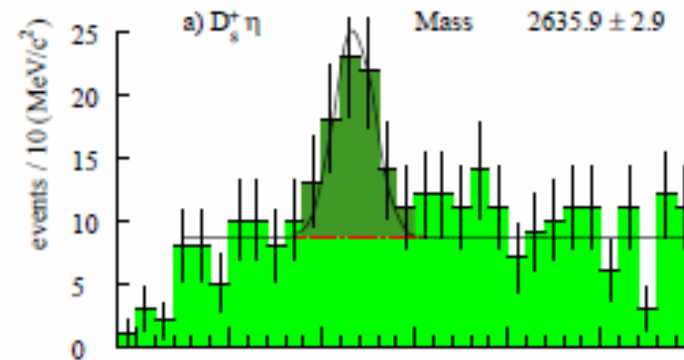
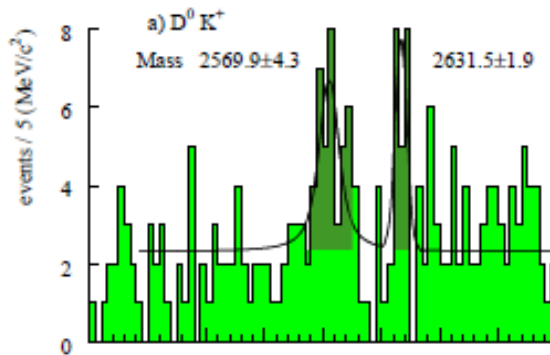
- $D_{sJ}(2860)$  (BaBar, PRL97, 222001 (2006))



$J^P = 0^+$   
2P state?

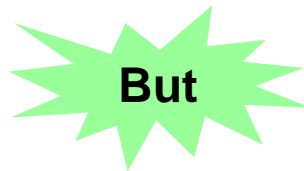
# SELEX $D_{sJ}(2632)$

- SELEX: charged hyperon beam at 600 GeV on Cu or C targets
- $D_{sJ}^+(2632)$ :  $2632.6 \pm 1.6$  MeV,  $\Gamma < 17$  MeV
  - Decay process:  $D_{sJ}(2632) \rightarrow D_s \eta$  ( $7.2 \sigma$ ),  $D^0 K^+$  ( $5.3 \sigma$ )



SELEX Collab., PRL93,242001(2004)

- $D_{sJ}^+(2632)$  might be radial excitation.    cf.  $J/\psi$  :  $M(2S) - M(1S) \approx 600$  MeV



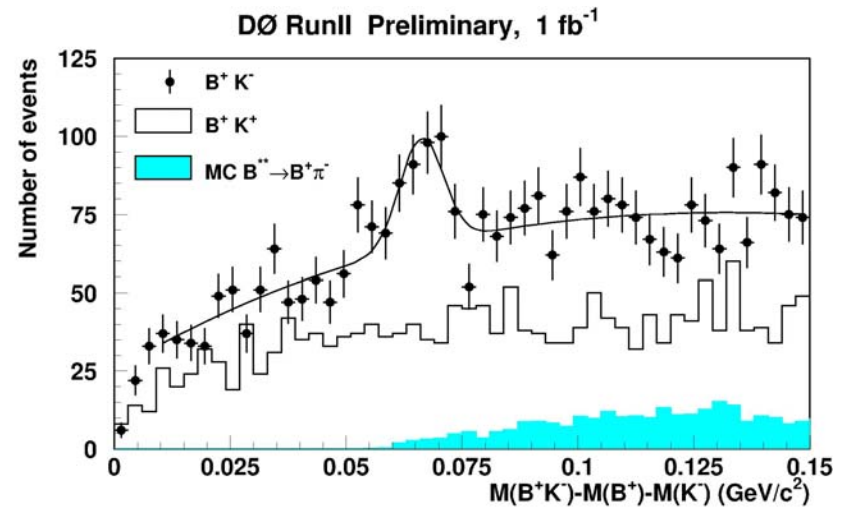
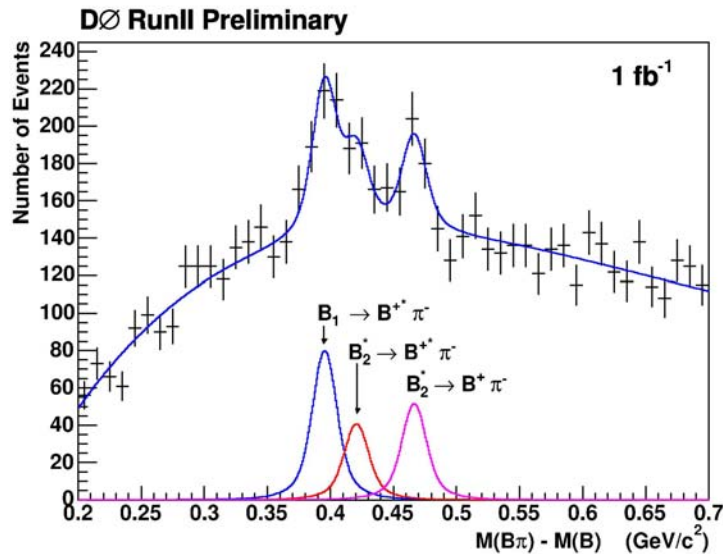
**BaBar/CLEO/Focus did not confirm  $D_{sJ}(2632)$ !!**  
 → Some people claim this might be a fake.

**$D_{sJ}(2632)$  is probably an experimental artifact.**



# New $B$ and $B_s$ Mesons

- $B_1(5720)[1^+]$ ,  $B_2^*(5745)[2^+]$ ,  $B_{s2}^*(5839)[2^+]$ 
  - CDF and D0, hep-ex/0605076



D. Gele, talk in DIS2006

- New heavier states will be discovered in LHC.



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# Hidden Charm Events

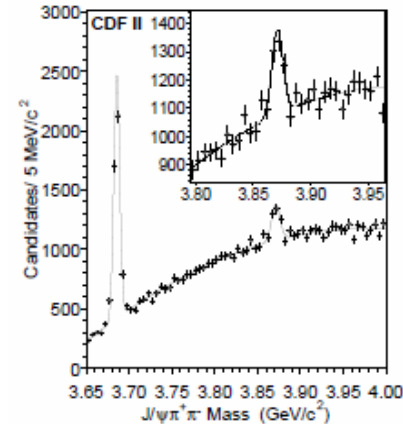
**$X(3872)$  ,  $X(3940)$**   
 **$Y(3940)$  ,  $Y(4260)$**   
 **$Z(3930)$**

# X(3872)

- Observed by Belle('04), and confirmed by CDFII, D0, BaBar('05)
- $M(X)=3871.9\pm 0.5$  MeV,  $\Gamma < 2.3$  MeV

Belle, BaBar  $B^\pm \rightarrow K^\pm X \rightarrow K^\pm \pi^+ \pi^- J/\psi$  ( $10\sigma$ )

CDFII, D0  $p\bar{p} \rightarrow X \rightarrow \pi^+ \pi^- J/\psi$  ( $11.6\sigma$ )



CDFII, PRL93, 072001(2004)

**Belle('05)**  $\frac{\Gamma(X \rightarrow \gamma J/\psi)}{\Gamma(X \rightarrow \pi^+ \pi^- J/\psi)} = 0.14 \pm 0.05 \quad \Rightarrow \quad C = + \text{ is established}$

$\frac{Br(X \rightarrow \pi^+ \pi^- \pi^0 J/\psi)}{Br(X \rightarrow \pi^+ \pi^- J/\psi)} = 1.0 \pm 0.4 \pm 0.3 \quad \Rightarrow \quad \text{strong isospin violation}$

- Angular correlation: rule out  $0^{++}, 0^{+-}$
- dilepton mass distribution: strongly disfavor  $1^{-+}, 2^{-+}$

- It is likely that X is a  $1^{++}$  charmonium state.



# X(3872) (cont.)

- Charmonium spectra
  - Swanson, hep-ph/0601110, Table II

Possible candidates {

- Experiments favor

$$\chi_1(2^3P_1) \dots 1^{++}$$

- Theoretical prediction of decay width is too much large.

- The others??

- hybrid, glueballs, tetraquark(diquark cluster), molecule,,,

State	PDG[16]	BGS[12]	GI[12]	EFG[13]	Cornell[3]	CP-PACS[14]	Chen[15]
$J/\psi(1^3S_1)$	$3096.87 \pm 0.04$	3090	3098	3096	3095 [3095]	$3085 \pm 1$	$3084 \pm 4$
$\eta_c(1^1S_0)$	$2979.2 \pm 1.3$	2982	2975	2979	3095	$3013 \pm 1$	$3014 \pm 4$
$\psi(2^3S_1)$	$3685.96 \pm 0.09$	3672	3676	3686	3684 [3684]	$3777 \pm 40$	$3780 \pm 43$
$\eta_c'(2^1S_0)$	$3637.7 \pm 4.4$	3630	3623	3588	3684	$3739 \pm 46$	$3707 \pm 20$
$\psi(3^3S_1)$	$4040 \pm 10$	4072	4100	4088	4110 [4225]	-	-
$\eta_c(3^1S_0)$		4043	4064	3991	4110	-	-
$\psi(4^3S_1)$	$4415 \pm 6$	4406	4450	-	4460 [4625]	-	-
$\eta_c(4^1S_0)$		4384	4425	-	4460	-	-
$\chi_2(1^3P_2)$	$3556.18 \pm 0.13$	3556	3550	3566	3522 [3523]	$3503 \pm 24$	$3488 \pm 11$
$\chi_1(1^3P_1)$	$3510.51 \pm 0.12$	3505	3510	3510	3522 [3517]	$3472 \pm 9$	$3462 \pm 15$
$\chi_0(1^3P_0)$	$3415.3 \pm 0.4$	3424	3445	3424	3522	3408	$3413 \pm 10$
$h_c(1^1P_1)$	$3524 \pm 1^a$	3516	3517	3526	3522 [3519]	$3474 \pm 40$	$3474 \pm 20$
$\chi_2(2^3P_2)$	$3931 \pm 5^a$	3972	3979	3972	-	$4030 \pm 180$	-
$\chi_1(2^3P_1)$		3925	3953	3929	-	$4067 \pm 105$	$4010 \pm 70$
$\chi_0(2^3P_0)$		3852	3916	3854	-	$4008 \pm 122$	$4080 \pm 75$
$h_c(2^1P_1)$		3934	3956	3945	-	$4053 \pm 95$	$3886 \pm 92$
$\chi_2(3^3P_2)$		4317	4337	-	-	-	-
$\chi_1(3^3P_1)$		4271	4317	-	-	-	-
$\chi_0(3^3P_0)$		4202	4292	-	-	-	-
$h_c(3^1P_1)$		4279	4318	-	-	-	-
$\psi_3(1^3D_3)$		3806	3849	3815	3810	-	$3822 \pm 25$
$\psi_2(1^3D_2)$		3800	3838	3811	3810	-	$3704 \pm 33$
$\psi(1^3D_1)$	$3769.9 \pm 2.5$	3785	3819	3798	3810 [3755]	-	-
$\eta_{c2}(1^1D_2)$		3799	3837	3811	3810	-	$3763 \pm 22$
$\psi_3(2^3D_3)$		4167	4217	-	4190	-	-
$\psi_2(2^3D_2)$		4158	4208	-	4190	-	-
$\psi(2^3D_1)$	$4159 \pm 20$	4142	4194	-	4190 [4230]	-	-
$\eta_{c2}(2^1D_2)$		4158	4208	-	4190	-	-
$\chi_4(1^3F_4)$		4021	4095	-	-	-	-
$\chi_3(1^3F_3)$		4029	4097	-	-	-	$4222 \pm 140$
$\chi_2(1^3F_2)$		4029	4092	-	-	-	-
$h_{c3}(1^1F_3)$		4026	4094	-	-	-	$4224 \pm 74$
$\chi_4(2^3F_4)$		4348	4425	-	-	-	-
$\chi_3(2^3F_3)$		4352	4426	-	-	-	-
$\chi_2(2^3F_2)$		4351	4422	-	-	-	-
$h_{c3}(2^1F_3)$		4350	4424	-	-	-	-
$\psi_5(1^3G_5)$		4214	4312	-	-	-	-
$\psi_4(1^3G_4)$		4228	4320	-	-	-	-
$\psi_3(1^3G_3)$		4237	4323	-	-	-	-
$\eta_{c4}(1^1G_4)$		4225	4317	-	-	-	-



# Theoretical Approaches

- **Quark (Potential) Model**  $c\bar{c}$ 
  - Large mass ( $\sim 100$  MeV above), Broad width
  - can not explain large isospin violating  $\rho J/\psi$  decay mode
- **Tetraquark:**  $cq\bar{c}\bar{q}$ 
  - Charged partner should sit around 3872 GeV.
  - But, no charged  $X$  states in experiments.
- **Hybrid:**  $c\bar{c}g$ 
  - $X \rightarrow J/\psi gg \rightarrow J/\psi \pi\pi$
  - Lattice & flux tube model predict  $M(c\bar{c}g) \approx 4200-4400$  MeV
- **Glueball:**  $gg$ 
  - Vector glueball with a small admixture of vector  $c\bar{c}$
- **Molecular State:**  $D\bar{D}^*$ 
  - Small binding energy  $D^0\bar{D}^{0*} = 3871.2 \pm 1.0$  MeV
  - Pion exchange at long range & quark/gluon exchange at short range
  - Can explain hidden charm decay mode:  $\rho J/\psi$ ,  $\omega J/\psi$
  - Cannot explain branching ratio

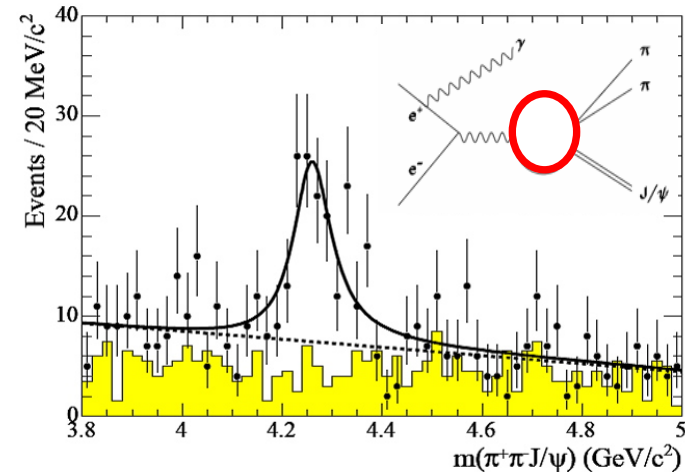
# Y(4260)

- Observed by BaBar('05), and confirmed by CLEO, Belle('06)
- $M(X)=4259 \pm 8 \text{ MeV}$ ,  $\Gamma=88 \pm 23 \text{ MeV}$

$$e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi \pi\pi$$

$$J^{PC} = 1^{--}$$

BaBar, PRL95,  
142001 (2005)



	BaBar	CLEO-c	CLEO III	Belle
N	$125 \pm 23$ ( $\sim 8\sigma$ )	$\sim 50$ ( $11\sigma$ )	$14.1 \pm 5.2$ ( $4.9\sigma$ )	$165 \pm 24$ ( $>7\sigma$ )
Mass (MeV)	$4259 \pm 8$	4260	$4283 \pm 17$	$4295 \pm 10$
Width (MeV)	$88 \pm 23$		$70 \pm 40$	$133 \pm 26$

- But, PDG  $1^-$  charmonium state is already occupied.



# What is $Y(4260)$ ?

---

- $1^-$  charmonium
  - **No suitable position** around this mass region (overpopulation)
  - Hidden charm decay width is large

One might expect comparable  $J/\psi\pi\pi$  width

$$\Gamma(\psi'' \rightarrow J/\psi \pi\pi) \approx 50 \text{ keV} \quad \text{vs} \quad \Gamma(Y \rightarrow J/\psi \pi\pi) > 1.8 \text{ MeV}$$

- **Molecule**
  - Close to  $\bar{D}D_1(2420)$ ,  $\bar{D}D_1^*$ ,  $\bar{D}^*D_0^*(2310)$  thresholds
  - **Possibility not excluded**
- **Tetraquark**
  - Decay into  $D\bar{D}$  easily  $\rightarrow$  Y's width would be much larger
  - Isospin-partner can decay into  $J/\psi\pi\pi\pi$  : **Ruled out by BaBar!**



# 1<sup>-</sup> Charmonium

- Predicted 4S charmonium at 4262 MeV exactly
  - Ding, Chao, Qin, PRD51, 5064 (1995)

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + Tr \left( \frac{1 - e^{-\mu r}}{\mu r} \right)$$

- PDG assignment is correct? PDG miss a 1<sup>-</sup> state?
- Challenge: How to explain large  $J/\psi\pi\pi$  decay width?

TABLE I. Calculated masses and leptonic widths for charmonium states with the screened potential (5) and parameters (8), where  $\Gamma_{ee} = \Gamma_{ee}^0 [1 - \frac{16}{3\pi} \alpha_s(m_c)]$  with  $\alpha_s(m_c) = 0.28$  [16].

States	Mass (MeV)	PDG	$\Gamma_{ee}$ (keV)	$\Gamma_{ee}^{\text{expt}}$ (keV)	Candidate
1S	3097	3097	5.34	$5.26 \pm 0.37$	$\psi(3097)$
2S	3686	3686	2.17	$2.14 \pm 0.21$	$\psi(3686)$
3S	4033	4040	1.23	$0.75 \pm 0.15$	$\psi(4040)$
4S	4262	4415	0.77	$0.77 \pm 0.23$	$\psi(4160)$
5S	4415		0.48	$0.47 \pm 0.10$	$\psi(4415)$
1P	3526				$\chi(3526)_{\text{c.o.g.}}$
1D	3805	3770			$\psi(3770)$
2D	4105	4160			



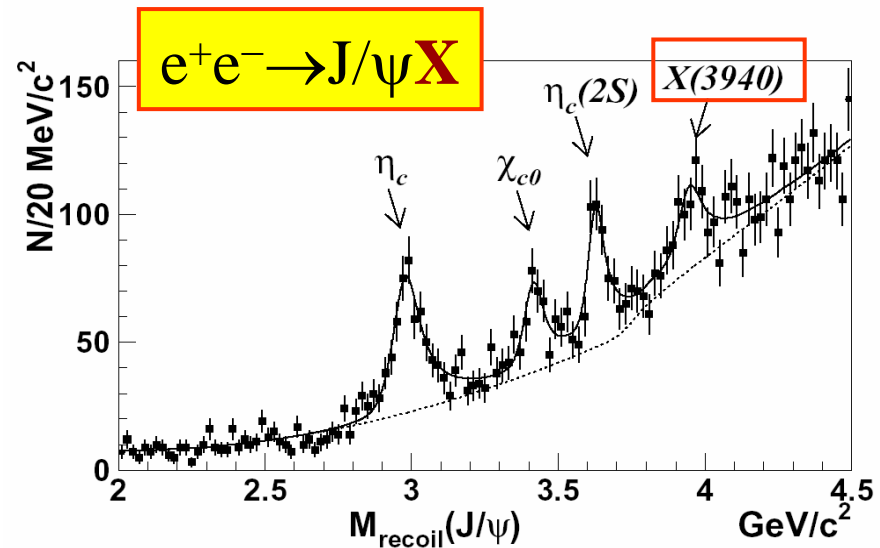
# X(3940)

- Observed by Belle('07)
- $M(X)=3936\pm 14$  MeV,  $\Gamma<52$  MeV,  $C=+$

Observed in  $\bar{D}D^*$  channel  
 Not seen in  $\bar{D}D$  and  $\omega J/\psi$



Probably  $\chi'_{c1}$  (2P) or  $\eta''_c$  (3S)



- But the ground state  $\chi_{c1}$ (3510) is not seen
- $\eta_c(3S)$  is another candidate of X(3940) except it's 100 MeV below QM prediction



# Summary of Hidden Charm Events

- $X(3872)$ ,  $\Gamma < 2.3$  MeV

$$X \rightarrow \rho J/\psi, \omega J/\psi$$

maybe  $\chi_{c1}'(2^3P_1)$

- $X(3940)$ ,  $\Gamma < 52$  MeV

$$X \rightarrow D\bar{D}^* \text{ not to } \omega J/\psi, D\bar{D}$$

maybe  $\chi_{c1}'(2^3P_1)$  or  $\eta_c''(3^1S_0)$

- $Y(3940)$ ,  $\Gamma \sim 87$  MeV

$$Y \rightarrow \omega J/\psi \text{ not to } D\bar{D}, D\bar{D}^*$$

Hybrid model strongly suppress  $D\bar{D}, D\bar{D}^*, D^*\bar{D}^*$  decay modes.  
But, mass less than 4000 MeV is **in conflict** with lattice calculation.

- $Z(3930)$ ,  $\Gamma \sim 20$  MeV

$$Z \rightarrow D\bar{D} \text{ to be consistent with } J=2 \text{ form decay helicity distribution}$$

Natural assignment is  $\chi_{c2}'$ .

- $Y(2460)$ ,  $\Gamma \sim 88$  MeV

$$Y \rightarrow J/\psi \pi\pi \text{ higher radial excitation, } \psi(4S), \psi(5S)??$$



---

# Relativistic Potential Model Approach within $1/m_Q$ Expansion

T. Matsuki, T. Morii, and K. Sudoh

## References:

- **New Heavy-Light Mesons  $Q\bar{q}$**   
Prog. Theor. Phys. 117, 1977 (2007).
- **$0^+$  and  $1^+$  States of  $B$  and  $B_s$  Mesons,**  
Phys. Lett. B606, 329 (2005).
- **Spectroscopy of heavy mesons expanded in  $1/m_Q$ ,**  
by T. M. and T. M., PRD56, 5646 (1997).



# Conventional Potential Model

- **Effective Hamiltonian** (Godfrey et al., PRD43, 1697 (1991))

$$H = H_{\text{free}} + H_{\text{int}}$$

$$H_{\text{free}} = (p^2 + m_q^2)^{1/2} + (p^2 + m_Q^2)^{1/2} : \text{Klein-Gordon}$$

$$H_{\text{int}} = \underbrace{H_{\text{conf+Coulomb}} + H_{\text{tensor}} + H_{\text{spin-orbit}}}$$

including many parameters

- Kinetic terms are Klein-Gordon (not spinors).
  - does not respect **heavy quark symmetry & chiral symmetry**.
- How do we incorporate **relativistic effects** in the system?
    - Succeeded in heavy quarkonium system
      - ➔ non-relativistic picture works well
    - No negative energy states both of  $q$  and  $Q$ .
      - ➔ degrees of freedom of wave function is reduced

$$4 \otimes 4 \xrightarrow{\text{FWT trans.}} 2 \otimes 2 \xrightarrow[\text{neglect negative part}]{\text{neglect}} 1 \otimes 1$$



# Strategy of Our Approach

- We propose a model with some improvements of the conventional potential model.
  - Construct the effective Hamiltonian with **chiral symmetry** and **heavy quark symmetry**
  - Treat quarks as Dirac particles
    - Light quark: 4-component spinor
    - Heavy quark: 2-component by Foldy transformation
  - **Hamiltonian and wave function are expanded in  $1/m_Q$**

$$H\psi_l = E^l\psi_l \quad H = H_{\text{FWT}} - m_Q = m_Q H_{-1} + H_0 + \frac{1}{m_Q} H_1 + \frac{1}{m_Q^2} H_2 + \dots$$

$$E^l = E_0^l + \frac{1}{m_Q} E_1^l + \frac{1}{m_Q^2} E_2^l + \dots$$

$$\psi_l = \psi_{l0} + \frac{1}{m_Q} \psi_{l1} + \frac{1}{m_Q^2} \psi_{l2} + \dots$$

- Include the contribution from both positive and negative components of the wave function



# Hamiltonian Density

- **Hamiltonian density:**

$$\mathcal{H}_0 = \int dx^3 \left[ q^{\dagger c}(x) (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) q^c(x) + Q^\dagger(x) (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) Q(x) \right]$$

$$\mathcal{H}_{\text{int}} = \int \int dx^3 dx'^3 \bar{q}^c(x) O_i q^c(x) V_i(x-x') \bar{Q}(x') O_i Q(x')$$

$$O_s = 1, V_s = S(r) \quad \text{for scalar confining potential}$$

$$O_v = \gamma_\mu, V_v = V(r) \quad \text{for vector Coulomb potential}$$

- **Wave function:**

$$|\psi\rangle = \int d^3x \int d^3y \psi_{\alpha\beta}(x-y) q_\alpha^{c\dagger}(x) Q_\beta^\dagger(y) |0\rangle$$

- **Eigenvalue equation:**

$$H\psi = (m_Q + \tilde{E})\psi \quad : \tilde{E} = E - m_Q$$

$$\downarrow \text{FWT}$$

$$(H_{\text{FWT}} - m_Q) \otimes \psi_{\text{FWT}} = \tilde{E} \psi_{\text{FWT}} \quad : O_q O_Q \otimes \psi_{\text{FWT}} = (O_q)_{\alpha\beta} (\psi_{\text{FWT}})_{\beta\gamma} (O_Q)_{\gamma\delta}$$



# Eigenvalue Equation

- **Fermi-Yang Eq.** - effective Hamiltonian for 2-body bound state

$$H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q S \\ + \left[ 1 - \frac{1}{2} \left\{ \vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n}) \right\} \right] V$$

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4}{3} \frac{\alpha_s}{r}$$

Semi-relativistic approach to heavy meson

→ Foldy-Wouthuysen-Tani (FWT) transformation  
to the heavy quark ~1/m<sub>Q</sub> expansion

- **Heavy Meson System**

$$H \psi_l = E^l \psi_l$$

$$H = H_{\text{FWT}} - m_Q = m_Q H_{-1} + H_0 + \frac{1}{m_Q} H_1 + \frac{1}{m_Q^2} H_2 + \dots$$

$$E^l = E_0^l + \frac{1}{m_Q} E_1^l + \frac{1}{m_Q^2} E_2^l + \dots$$

$$\psi_l = \psi_{l0} + \frac{1}{m_Q} \psi_{l1} + \frac{1}{m_Q^2} \psi_{l2} + \dots$$



# Foldy Transformation

- **FWT transformation:**

- describe a heavy quark non-relativistically
- Degrees of freedom for  $Q$  and  $\bar{Q}$  independently conserve.

$$H_{\text{FWT}} = U_c U_{\text{FWT}}(p'_Q) H U_{\text{FWT}}^{-1}(p_Q) U_c^{-1}$$

$$\psi_{\text{FWT}} = U_c U_{\text{FWT}}(p_Q) \psi$$

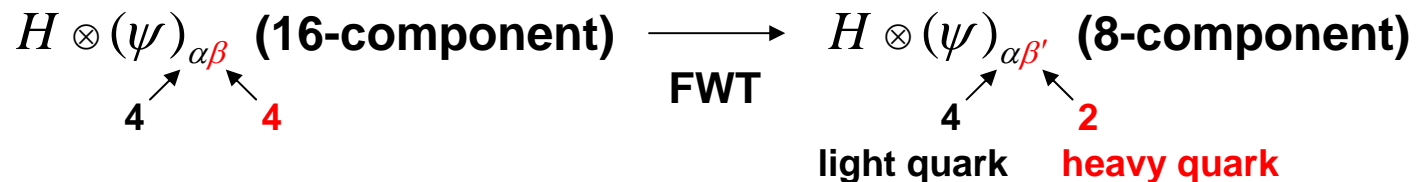
$$: U_{\text{FWT}}(p) = \exp(W(p) \vec{\gamma}_Q \cdot \vec{\hat{p}}) = \cos W + \vec{\gamma}_Q \cdot \vec{\hat{p}} \sin W$$

$$\vec{\hat{p}} = \frac{\vec{p}}{p}, \quad \tan W(p) = \frac{p}{m_Q + E}, \quad E = \sqrt{\vec{p}^2 + m_Q^2}$$

$$: U_c = i\gamma_Q^0 \gamma_Q^2 = -U_c^{-1}$$

- **Heavy quark symmetry** ~ in a limit  $m_Q \rightarrow \infty$

- **2-particles system**





# Effective Hamiltonian Expanded in $1/m_Q$

$$(H_{\text{FWT}} - m_Q) \otimes \psi_{\text{FWT}} = \tilde{E} \psi_{\text{FWT}}$$

$$H_{\text{FWT}} - m_Q = H_{-1} + H_0 + H_1 + H_2$$

$$H_{-1} = -(1 + \beta_Q) m_Q$$

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + \beta_q m_q - \beta_q \beta_Q S + \left\{ 1 + \frac{1}{2} [\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})] \right\} V$$

$$H_1 = -\frac{1}{2m_Q} \beta_Q \vec{p}^2 + \frac{1}{m_Q} \beta_q \vec{\alpha}_Q \cdot \left( \vec{p} + \frac{1}{2} \vec{q} \right) S + \frac{1}{2m_Q} \vec{\gamma}_Q \cdot \vec{q} V$$

$$-\frac{1}{2m_Q} \left[ \beta_Q \left( \vec{p} + \frac{1}{2} \vec{q} \right) + i \vec{q} \times \beta_Q \vec{\Sigma}_Q \right] \cdot \left[ \vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n}) \vec{n} \right] V$$

$$H_2 = \frac{1}{2m_Q} \beta_q \beta_Q \left( \vec{p} + \frac{1}{2} \vec{q} \right)^2 S - \frac{i}{4m_Q^2} \vec{q} \times \vec{p} \cdot \beta_q \beta_Q \vec{\Sigma}_Q S - \frac{1}{8m_Q^2} \vec{q}^2 V - \frac{i}{4m_Q^2} \vec{q} \times \vec{p} \cdot \vec{\Sigma}_Q V$$

$$-\frac{1}{8m_Q^2} \left\{ (\vec{p} + \vec{q})(\vec{\alpha}_Q \cdot \vec{p}) + \vec{p} [\vec{\alpha}_Q \cdot (\vec{p} + \vec{q})] + i \vec{q} \times \vec{p} \gamma_Q^5 \right\} \cdot \left[ \vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n}) \vec{n} \right] V$$

$$\because \vec{p} = \vec{p}_q = -\vec{p}_Q, \quad \vec{p}' = \vec{p}'_q = -\vec{p}'_Q, \quad \vec{q} = \vec{p}' - \vec{p}$$



# Structure of Mass Level

- We treat quarks as relativistic as possible and consistently takes into account **chiral & heavy quark symmetry**.

**Semi-relativistic approach**

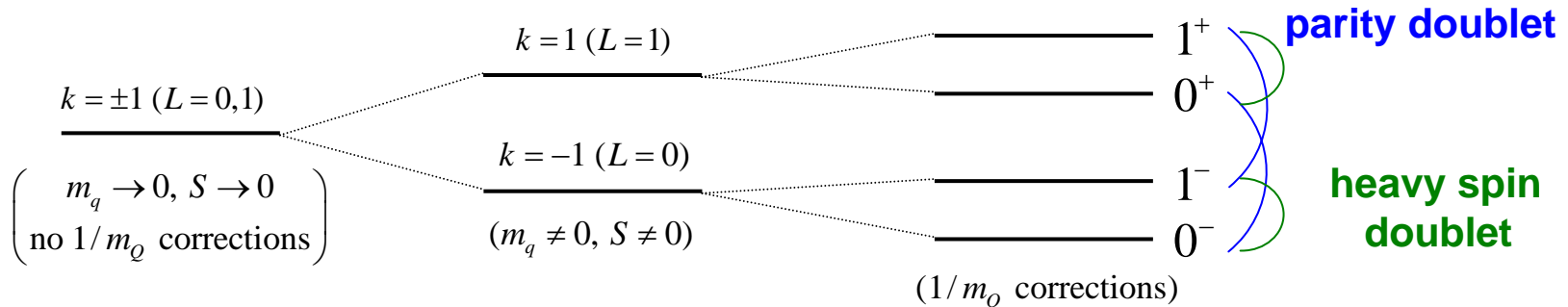
quark: relativistic, 4-spinor particle

heavy quark: (semi-)relativistic

**+  $1/m_Q$  corrections:** relativistic effects of heavy quark

- Negative energy states both of  $q$  and  $Q$  are included.

- Structure of mass level**



**Chiral symmetry is broken.**

**Heavy quark symmetry is broken.**



# Analysis in Variational Method

- **Input values to determine parameters**
  - 6  $D$  meson masses
  - 6  $D_s$  meson masses
  - $B$  and  $B_s$  meson masses are not input in a first step.
- **Most optical values of parameters**

Parameters	$\alpha_s$	$a$ (GeV <sup>-1</sup> )	$b$ (GeV)	$m_{u,d}$ (GeV)	$m_s$ (GeV)	$m_c$ (GeV)	$m_b$ (GeV)
first order	0.2610	1.939	0.0749	0.0112	0.0929	1.032	4.639
second order	0.3066	2.610	0.0999	0.0200	0.0825	1.173	4.668

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r}, \quad S(r) = \frac{r}{a^2} + b \quad \text{trial wave function: } \sim w_k(r) \left(\frac{r}{a}\right)^\gamma \exp\left[-(m_q + b)r - \frac{1}{2}\left(\frac{r}{a}\right)^2\right]$$

**Using above parameters, other mass levels are calculated up to  $1/m_Q^2$ .**

TABLE II:  $D$  meson mass spectra (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	1784	$0.476 \times 10^{-1}$	1869	1867
$^3S_1$	-1	$1^-$		$1.271 \times 10^{-1}$	2011	2008
$^3P_0$	1	$0^+$	2067	$1.046 \times 10^{-1}$	2283	2308
" $^3P_1$ "	1	$1^+$		$1.713 \times 10^{-1}$	2421	2427
" $^1P_1$ "	-2	$1^+$	2125	$1.415 \times 10^{-1}$	2425	2420
$^3P_2$	-2	$2^+$		$1.618 \times 10^{-1}$	2468	2460
$^3D_1$	2	$1^-$	2322	$1.894 \times 10^{-1}$	2762	—
" $^3D_2$ "	2	$2^-$		$2.054 \times 10^{-1}$	2800	—

TABLE III:  $D_s$  meson mass spectra (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	1900	$0.352 \times 10^{-1}$	1967	1969
$^3S_1$	-1	$1^-$		$1.102 \times 10^{-1}$	2110	2112
$^3P_0$	1	$0^+$	2095	$1.101 \times 10^{-1}$	2325	2317
" $^3P_1$ "	1	$1^+$		$1.778 \times 10^{-1}$	2467	2460
" $^1P_1$ "	-2	$1^+$	2239	$1.274 \times 10^{-1}$	2525	2535
$^3P_2$	-2	$2^+$		$1.467 \times 10^{-1}$	2568	2572
$^3D_1$	2	$1^-$	2342	$2.031 \times 10^{-1}$	2817	—
" $^3D_2$ "	2	$2^-$		$2.196 \times 10^{-1}$	2856	—

TABLE V:  $D$  meson mass spectra (second order). Units are in MeV.

State ( $^{2S+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_1$	$c_2/M_0$	$M_2$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	1783	$0.678 \times 10^{-1}$	1904	$-2.091 \times 10^{-2}$	1867	1867
$^3S_1$	-1	$1^-$		$1.245 \times 10^{-1}$	2005	$2.507 \times 10^{-3}$	2009	2008
$^3P_0$	1	$0^+$	1935	$1.694 \times 10^{-1}$	2262	$1.567 \times 10^{-2}$	2293	2308
$^3P_1$	1	$1^+$		$2.132 \times 10^{-1}$	2347	$1.665 \times 10^{-3}$	2350	2427
$^1P_1$	-2	$1^+$	2045	$1.866 \times 10^{-1}$	2427	$2.499 \times 10^{-3}$	2432	2420
$^3P_2$	-2	$2^+$		$1.980 \times 10^{-1}$	2450	$-1.003 \times 10^{-3}$	2448	2460
$^3D_1$	2	$1^-$	2127	$3.175 \times 10^{-1}$	2802	$2.109 \times 10^{-4}$	2803	-
$^3D_2$	2	$2^-$		$2.828 \times 10^{-1}$	2729	$-1.409 \times 10^{-3}$	2726	-

 TABLE VI:  $D_s$  meson mass spectra (second order). Units are in MeV.

State ( $^{2S+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_1$	$c_2/M_0$	$M_2$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	1873	$0.713 \times 10^{-1}$	2007	$-2.024 \times 10^{-2}$	1969	1969
$^3S_1$	-1	$1^-$		$1.249 \times 10^{-1}$	2107	$2.180 \times 10^{-3}$	2112	2112
$^3P_0$	1	$0^+$	1964	$1.685 \times 10^{-1}$	2294	$1.640 \times 10^{-2}$	2327	2317
$^3P_1$	1	$1^+$		$2.146 \times 10^{-1}$	2385	$2.211 \times 10^{-3}$	2389	2460
$^1P_1$	-2	$1^+$	2133	$1.916 \times 10^{-1}$	2542	$2.634 \times 10^{-3}$	2547	2535
$^3P_2$	-2	$2^+$		$2.030 \times 10^{-1}$	2566	$8.387 \times 10^{-4}$	2568	2572
$^3D_1$	2	$1^-$	2146	$3.315 \times 10^{-1}$	2857	$8.309 \times 10^{-4}$	2858	-
$^3D_2$	2	$2^-$		$2.751 \times 10^{-1}$	2736	$-1.329 \times 10^{-3}$	2733	-



# B and B<sub>s</sub> Mesons

- Include recently observed  $B(1^+)$ ,  $B(2^+)$  and  $B_s(2^+)$ 
  - determine  $m_b$
- Parameter  $\alpha_s$  is slightly modified
  - Actually  $\alpha_s$  is running with a momentum

Parameters	$\alpha_s^c$	$\alpha_s^b$	$a(\text{GeV}^{-1})$	$b(\text{GeV})$	$m_{u,d}(\text{GeV})$	$m_s(\text{GeV})$	$m_c(\text{GeV})$	$m_b(\text{GeV})$
	0.261	0.393	1.939	0.0749	0.0112	0.0929	1.032	4.639

- Typical energy scale is given by  $\bar{\Lambda}$  parameter in HQET
  - A kind of binding energy

$$\bar{\Lambda} \equiv \lim_{m_Q \rightarrow \infty} (M_{\text{calc}} - m_Q) \quad \bar{\Lambda}_u \equiv M_0(D : k = -1) - m_c = \mathbf{752} \text{ MeV for } D$$

$$\bar{\Lambda}_u \equiv M_0(B : k = -1) - m_b = \mathbf{638} \text{ MeV for } B$$

- Similar tendency for  $D_s$  and  $B_s$
- In HQET,  $\bar{\Lambda} \sim$  residual momentum  $k$ ;  $p = m_Q v + k$ , not  $m_Q$

TABLE IV:  $B$  meson mass spectra (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	5277	$-0.161 \times 10^{-2}$	5270	5279
$^3S_1$	-1	$1^-$		$0.981 \times 10^{-2}$	5329	5325
$^3P_0$	1	$0^+$	5570	$0.400 \times 10^{-2}$	5592	—
$^3P_1^n$	1	$1^+$		$1.411 \times 10^{-2}$	5649	—
$^1P_1^n$	-2	$1^+$	5660	$1.069 \times 10^{-2}$	5720	5720(?)
$^3P_2$	-2	$2^+$		$1.363 \times 10^{-2}$	5737	5745(?)
$^3D_1$	2	$1^-$	5736	$2.201 \times 10^{-1}$	6999	—
$^3D_2^n$	2	$2^-$		$1.429 \times 10^{-1}$	6556	—

 TABLE V:  $B_s$  meson mass spectra (first order). Units are in MeV.

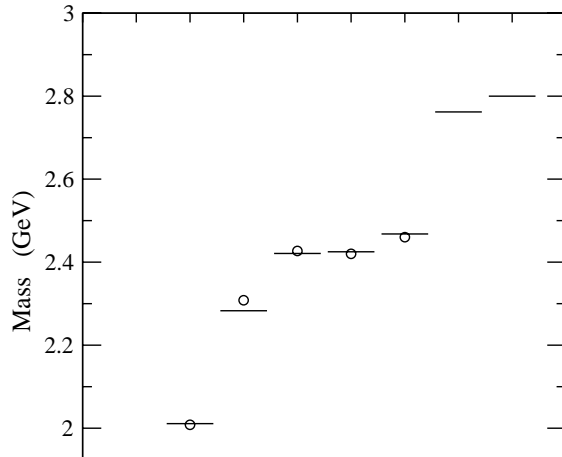
State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	5394	$-0.302 \times 10^{-2}$	5378	5369
$^3S_1$	-1	$1^-$		$0.853 \times 10^{-2}$	5440	—
$^3P_0$	1	$0^+$	5598	$0.350 \times 10^{-2}$	5617	—
$^3P_1^n$	1	$1^+$		$1.498 \times 10^{-2}$	5682	—
$^1P_1^n$	-2	$1^+$	5775	$0.978 \times 10^{-2}$	5831	—
$^3P_2$	-2	$2^+$		$1.263 \times 10^{-2}$	5847	5839(?)
$^3D_1$	2	$1^-$	5875	$2.949 \times 10^{-2}$	6048	—
$^3D_2^n$	2	$2^-$		$0.564 \times 10^{-2}$	5908	—



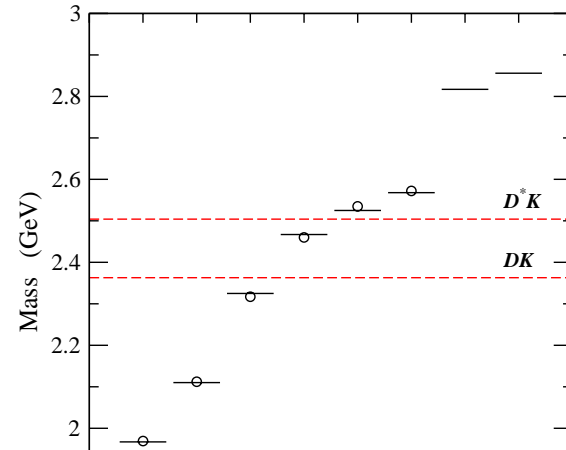
# Mass Spectra in 1st Order of $1/m_Q$

13/04/2011

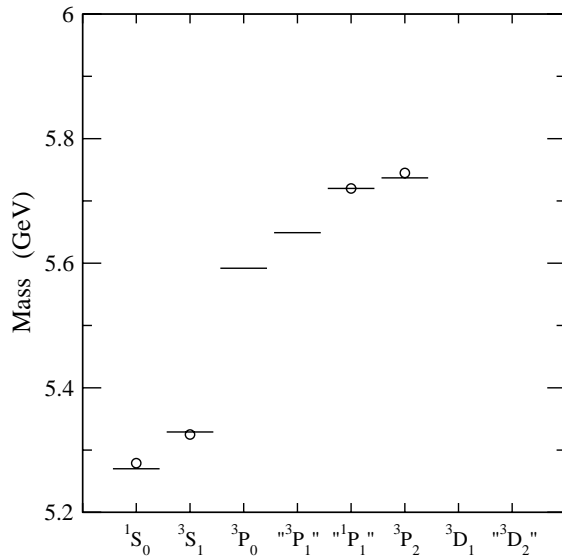
**D**



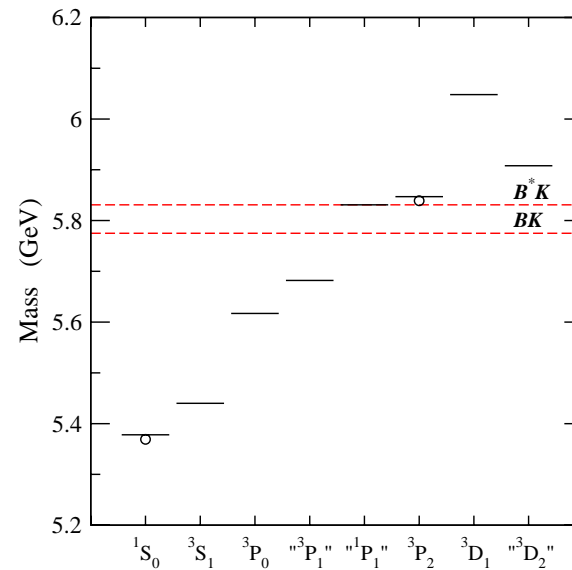
**D<sub>s</sub>**



**B**



**B**





# Radial Excitation

- New  $D_{sJ}$  particles were observed.
  - $D_{sJ}(2715)$  : from Belle (hep-ex/0608031)
  - $D_{sJ}(2856)$  : from BaBar (hep-ex/0607082)
- These states might be radial excitation ( $n=2$ ).
- We estimate masses of radial excitation using parameters which are obtained by  $n=1$  analyses.  
(but only  $\alpha_s$  is different)

TABLE I: Parameter set which is obtained by the previous analysis.

Parameters	$\alpha_s^{n=2}$	$a$ (GeV $^{-1}$ )	$b$ (GeV)	$m_{u,d}$ (GeV)	$m_s$ (GeV)	$m_c$ (GeV)	$m_b$ (GeV)
	0.344	1.939	0.0749	0.0112	0.0929	1.032	4.639



# $D$ and $D_s$ : $n=2$

TABLE II:  $n = 2$  radial excitation spectra of  $D$  meson (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	2413	$1.078 \times 10^{-1}$	2483	—
$^3S_1$	-1	$1^-$		$1.917 \times 10^{-1}$	2671	—
$^3P_0$	1	$0^+$	2418	$1.540 \times 10^{-1}$	2791	—
" $^3P_1$ "	1	$1^+$		$2.493 \times 10^{-1}$	3021	—
" $^1P_1$ "	-2	$1^+$	2491	$2.076 \times 10^{-1}$	3008	—
$^3P_2$	-2	$2^+$		$2.319 \times 10^{-1}$	3069	—

TABLE III:  $n = 2$  radial excitation spectra of  $D_s$  meson (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	2328	$1.006 \times 10^{-1}$	2563	—
$^3S_1$	-1	$1^-$		$1.830 \times 10^{-1}$	2755	2715(?)
$^3P_0$	1	$0^+$	2456	$1.553 \times 10^{-1}$	2837	2856(?)
" $^3P_1$ "	1	$1^+$		$2.551 \times 10^{-1}$	3082	—
" $^1P_1$ "	-2	$1^+$	2585	$1.969 \times 10^{-1}$	3094	—
$^3P_2$	-2	$2^+$		$2.209 \times 10^{-1}$	3157	—



# $B$ and $B_s$ : $n=2$

TABLE IV:  $n = 2$  radial excitation spectra of  $B$  meson (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	5849	$0.919 \times 10^{-2}$	5902	—
$^3S_1$	-1	$1^-$		$1.634 \times 10^{-2}$	5944	—
$^3P_0$	1	$0^+$	6025	$1.375 \times 10^{-2}$	6108	—
" $^3P_1$ "	1	$1^+$		$2.226 \times 10^{-2}$	6160	—
" $^1P_1$ "	-2	$1^+$	6098	$1.886 \times 10^{-2}$	6213	—
$^3P_2$	-2	$2^+$		$2.108 \times 10^{-2}$	6227	—

TABLE V:  $n = 2$  radial excitation spectra of  $B_s$  meson (first order). Units are in MeV.

State ( $^{2s+1}L_J$ )	$k$	$J^P$	$M_0$	$c_1/M_0$	$M_{\text{calc}}$	$M_{\text{obs}}$
$^1S_0$	-1	$0^-$	5936	$0.878 \times 10^{-2}$	5988	—
$^3S_1$	-1	$1^-$		$1.597 \times 10^{-2}$	6031	—
$^3P_0$	1	$0^+$	6063	$1.399 \times 10^{-2}$	6148	—
" $^3P_1$ "	1	$1^+$		$2.299 \times 10^{-2}$	6202	—
" $^1P_1$ "	-2	$1^+$	6193	$1.828 \times 10^{-2}$	6306	—
$^3P_2$	-2	$2^+$		$2.052 \times 10^{-2}$	6329	—



# Summary

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- **We treated  $Q\bar{q}$  system in a semi-relativistic approach.**
  - Hamiltonian and wave functions are expanded in  $1/m_Q$ .
  - Masses are calculated up to 2nd order:  $1/m_Q^2$ .
- **Masses in our model are in good agreement within 1% accuracy.**
  - Degeneracy between each heavy spin multiplet  $(0^-, 1^-)$  and  $(0^+, 1^+)$  is automatically derived.
  - We expect these states will be measured in future experiments.
- **Accidental flavor  $SU(3)_f$  symmetry is realized for  $L=1$  states.**
  - We predict same phenomena for  $B$  and  $B_s$  mesons.
- **Mass spectra of radial excitation ( $n=2$ ) are estimated.**
- **Open a new way to calculate heavy mesons/baryons.**  
**Lots more coming.**