格子QCD計算によるさまざまなカラー自由度に依存したクォーク間ポテンシャルの研究

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PLB621(2005)171-175

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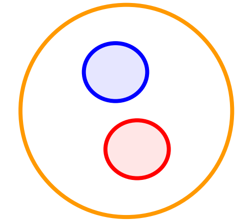
論文作成中

1. QCDカラーポテンシャル
2. 閉じ込め：クーロンゲージQCD
3. ポリヤコフループ相関関数
4. 非カラー一重項の発散
5. 数値計算結果
6. まとめ

QCD color potentials between two quarks

Color-singlet 1

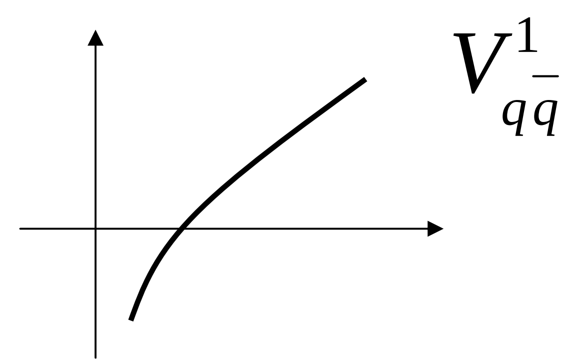


- Attraction. Strongest force between two quarks. Important for color confinement and/or color-singlet hadron bound state.
- Linearly rising potentials in the hadron phase.
- There are much many lattice studies. See a ref. Bali, PR343(2001) .

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$C = \langle \lambda_i \lambda_j \rangle = -\frac{4}{3}$$

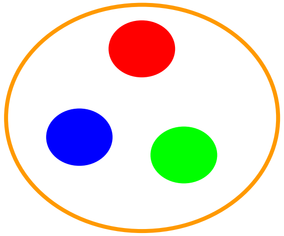
$$V_1 \sim \frac{C}{R}$$ at short distances



at long distances

◆ However, the gauge invariant Wilson loop or the gauge invariant Polyakov loop can not distinguish the color-singlet and color-octet contributions.

Color anti-symmetric 3*

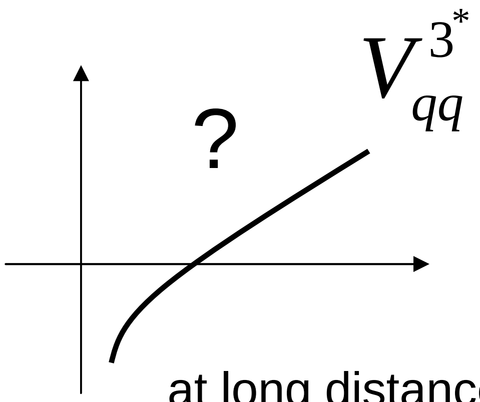


- Attraction. Di-quark. Very long history.
  (See a Review: Anselmino, et. al, RMP65(1995)1199.)
- Multi-quark state (reported by LEPS group), in which highly correlated di-quark ? Jaffe and Wilczek, PRL95(2003)232003
- Hybrid, exotic particles ?
- Di-quark condensate in the finite chemical potential.
  - Linearly rising potentials in the hadron phase ? Its strength ?
  - No lattice calculation for the color anti-triplet (di-quark) potential.

$$3 \otimes 3 = 6 \oplus \bar{3}$$

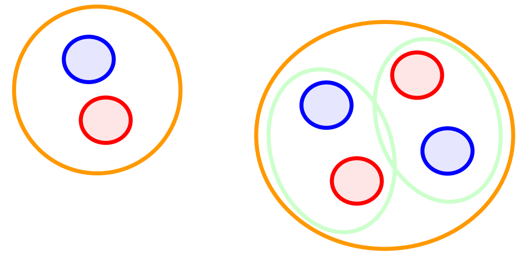
$$C = \langle \lambda_i \lambda_j \rangle = -\frac{2}{3}$$

$$V_{3^*} \sim \frac{C}{R}$$ at short distances



at long distances

Color octet 8



- Repulsion. Weakest force between two quarks.

- Multi-quark, hybrid, exotics.

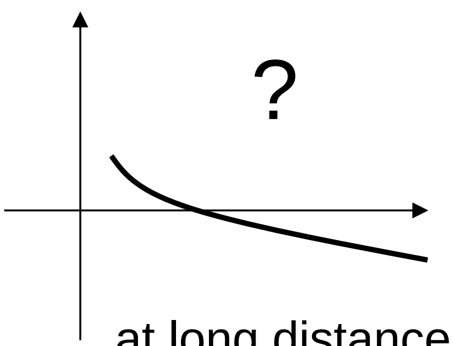
- Precise analyses of J/Ψ photo-productions, in which color-octet model is important.
Experiment：CLEO Collab. PRD70,072001
Theories：Cacciari and Kramer, PRL76,4128(1999), Bratten and Fleming, PRL74,3327(1995), etc.

◆ No lattice calculation of this potential.

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$C = \langle \lambda_i \lambda_j \rangle = \frac{1}{6}$$

$$V_8 \sim \frac{C}{R}$$ at short distances



at long distances

Our aim in this study

- Main aim
  - We'd like to clarify the long-distance behavior of color-dependent forces between two quarks in the lattice QCD simulation. Finally, we hope that this result may improve a model study of hadrons.
- Today's topic
  - Color forces depend on the color (Casimir) factor at large distances?
  - Volume dependence due to the divergence of color-flux in the color non-singlet channels ?
- Method
  - Here we employ the color-decomposed Polyakov line correlators in the Coulomb gauge QCD.
  - Moreover, we focus instantaneous potentials of QCD in the hadron phase.

Confinement:

Coulomb gauge confinement scenario

Study of confinement

- In order to study the internal structure of hadrons, we have to know how confining quark potentials behave in the confining phase.
- But, it's very difficult to understand color confinement, which is one of the most challenging issues in particle and nuclear physics.
- There were several approaches and a lot of works to understand the confinement …. :
  - ✓ Dual superconductor scenario, centre vortex model, the measurements of gluon, quark, ghost propagators in infrared regions, etc.
  - ✓ Topological quantities in the QCD vacuum are important : *magnetic monopole, instanton, centre vortex, etc.*
  - ✓ We should use a proper gauge fixing to realize these properties.

Coulomb gauge confinement

- In our study, we use the Coulomb gauge QCD, which has several good properties:
  - Coulomb gauge is a physical gauge.
  - Transverse mode of gluon propagators.
  - No indefinite metric; negative spectral function will not appear.
  - Instantaneous potential that can be defined in the Coulomb gauge is important to study the quark bound state, hadrons. (This is somewhat the analogy of QED.) This gives us the basic properties of color interactions among quarks.
- In this framework, we try to calculate the long-distance instantaneous part to investigate color-dependent forces in the lattice QCD simulation.

Coulomb gauge QCD

(Gribov-Zwanziger scenario)

Coulomb gauge QCD

- Hamiltonian in the Coulomb gauge QCD

$$H=\frac{1}{2}\int d^3x\left(E_i^2+B_i^2\right)+\frac{1}{2}\int d^3x d^3y\left(\rho(x)D(x,y)\rho(y)\right)$$

- Faddeev-Popov term in the Coulomb gauge QCD

$$D(\vec{x},\vec{y})=\int d^3z\left[\frac{1}{M(\vec{x},\vec{y})}(-\vec{\partial}_{\vec{z}}^2)\frac{1}{M(\vec{x},\vec{y})}\right] \quad M=-(\vec{\partial}^2+g\vec{A}\times\vec{\partial})$$

- Time-time component of the gluon propagators.

vacuum polarization (retarded) part

$$g^2\left\langle A_0(x)A_0(y)\right\rangle=V(x-y)+P(x-y)$$

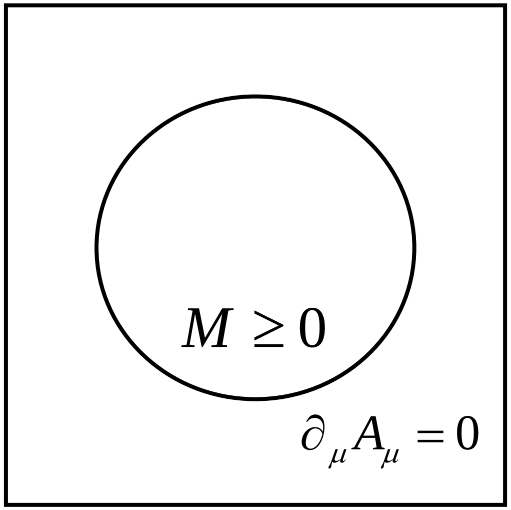
Instantaneous part $$V(x-y)=g^2\left\langle D(\vec{x},\vec{y})\right\rangle\delta(x_4-y_4)$$

Fadeev-Popov and instant. parts

Instantaneous part is defined in terms of FP operator in QCD

$$V_{inst}(r) \Rightarrow \left\langle \frac{1}{M}\left(-\partial_i^2\right)\frac{1}{M} \right\rangle, \text{ M: Fadeev-Popov operator}$$

- Gribov conjecture (or example)
  - ✓ Low-lying mode of the eigenvalues of FP cause the singular behavior of a confining potential.
  - ✓ Namely, their low-lying mode is responsible for the color confinement.
  - ✓ This is necessary to realize the confinement.



$M \geq 0$; Gribov region

Zwanziger's inequality

" No confinement without Coulomb confinement "

Zwanziger, PRL90, 102001 (2003)

$$V_{phys}(R) \le V_{coul}(R)$$

Here the physical potential corresponds to the Wilson loop potential.

If the physical potential is confining, then the color-Coulomb potential is also confining.

This is a necessary condition for color confinement.

Related refs. on the Coulomb gauge QCD (1)

Original idea

1. Study of confinement by Gribov. NPB139,1 (1978)
2. Color-Coulomb instantaneous part is very important, which is advocated by Zwanziger, NPB518,237 (1998)
3. Study of the renormalization of the Coulomb gauge QCD, Baulieu and Zwanziger, NPB548,527(1998); A.Niegawa, PRD74,045021(2006),A.Niegawa,M.Inui and H.Kohyama, PRD74,105016(2006)

Lattice calculations

1. The SU(2) lattice simulation proved that the infrared part, D_{00}(k=0), shows the large contributions, while the spatial part D_{ii} (k=0) is suppressed. Cucchieri, Zwanziger, PRD65,0142002,(2002)
2. There is an inequality between color-Coulomb and physical potentials, which is found by Zwanziger, PRL90, 102001 (2003)

Related refs. on the Coulomb gauge QCD (2)

Lattice calculations

3. The SU(2) lattice simulation shows that the instantaneous part is confining potential.
   Greensite, Olejnik, PRD67,094503(2003); PRD69,074506(2004).
4. The SU(3) lattice simulation shows similar numerical result for the color-Coulomb instantaneous part.
   Nakamura, Saito, PTP115(2006)189-200.
5. Recently, in the QGP phase, even at T/T_c~5.0, the linearity of the color-Coulomb potential persists. We discuss the relation between the color-Coulomb string tension and the magnetic scaling.
   Nakagawa, Nakamura, Saito, Toki, Zwanziger, PRD73 (2006) 094504.
6. QCD color interactions between two quarks
   Nakamura and Saito PLB621(2005)171.
7. Most recently, the role of the lowest-mode of SU(3) FP operator has been discussed. Nakagawa, Nakamura, Saito, Toki, PRD75(2007)014508.

Related refs. on the Coulomb gauge QCD (3)

Applications to hadron physics

1. Variational solution of the Yang-Mills Schrodinger equation in Coulomb gauge,
   Feuchter and Reinhardt, PRD70 (2004) 105021.
2. Yang-Mills wave functional in Coulomb gauge
   Reinhardt, Feuchter, PRD71 (2005) ,105002.
3. Aspects of the confinement mechanism in Coulomb-gauge QCD
   Alkofer, Kloker, Krassnigg, and Wagenbrunn PRL96 (2006) 022001.
4. Coulomb gauge model of mesons
   Ligterink and Swanson, PRC69, 025204 (2004).
5. Coulomb gauge QCD, confinement, and the constituent representation
   Szczepaniak and Swanson, PRD65, 025012 (2001)

Related refs. on the Coulomb gauge QCD (4)

Applications to hadron physics

6. Confinement and gluon propagator in Coulomb gauge QCD
   Szczepaniak, PRD69, 074031 (2004).
7. Chromoelectric flux tubes
   Bowman and Szczepaniak, PRD70, 016002 (2004).
8. Coulomb energy and gluon distribution in the presence of static sources
   Szczepaniak and Krupinski, PRD73, 034022 (2006).
9. QCD green functions and their application to hadron physics
   Alkofer, hep-ph/0611090
10. Mesons and diquarks in Coulomb-gauge QCD
    Wagenbrunn, hep-ph/0610400

Related refs. on the Coulomb gauge QCD (5)

Applications to QGP physics

1. Equation of state of gluon plasma from a fundamental modular region
   Zwanziger, PRL94, 182301 (2005)
2. Equation of state of gluon plasma from Local action.
   Zwanziger, hep-ph/0610021.

Polyakov loop correlations

Polyakov line

- Polyakov line (McLerran, RMP58, 1021(1986))

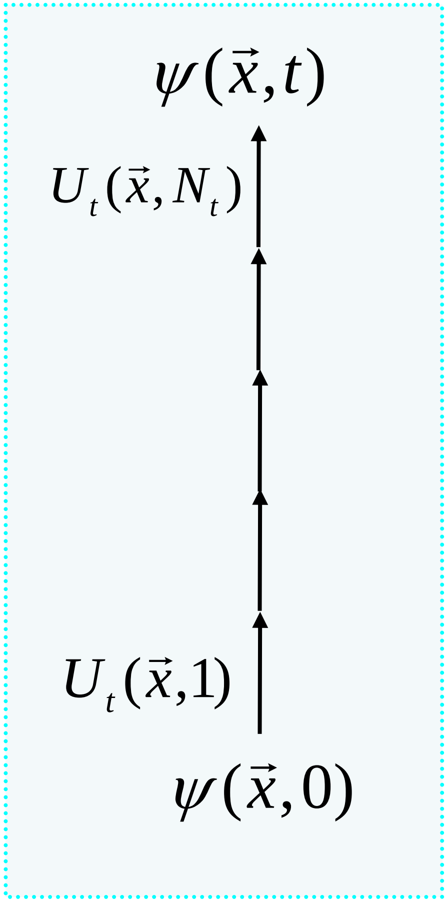
$$\left(\frac{1}{i}\frac{\partial}{\partial t} - t^a A_0^a(\vec{x},t)\right)\psi(\vec{x},t) = 0$$

$$\psi(\vec{x},t) = T\exp\left(i\int_0^t dt' t^a A_0^a(\vec{x},t')\right)\psi(\vec{x},0)$$

$$= L(\vec{x})\psi(\vec{x},0)$$

$$L(\vec{x}) \equiv U_t(\vec{x},N_t)U_0(\vec{x},N_t-1)\ldots U_0(\vec{x},1)$$

$$U_\mu(x) = \exp(iagA_\mu(x))$$



Quark-antiquark potential

- Color-average

  $3 \otimes \bar{3} = 1 \oplus 8$

Nadkarni, PRD33,3738 (1986)

$$\left\langle \mathrm{TrL(R)TrL^{+}(0)} \right\rangle = 1 \cdot \exp(-N_t V_1(R)) + (N^2 - 1) \cdot \exp(-N_t V_8(R))$$

- Color-singlet（attractive）

$$\mathrm{e}^{-V_1(R)} = \frac{1}{3} \left\langle \mathrm{TrL(R)L^{\dagger}(0)} \right\rangle \quad C = -\frac{4}{3}$$

$$V_i \sim \frac{C}{R}$$

- Color-octet（repulsive）

$$\mathrm{e}^{-V_8(R)} = \frac{8}{9} \left\langle \mathrm{TrL(R)TrL^{+}(0)} \right\rangle - \frac{3}{8} \left\langle \mathrm{TrL(R)L^{+}(0)} \right\rangle \quad C = +\frac{1}{6}$$

Quark-quark potential

- Color average

Nadkarni, PRD34,3904,1988

$$3 \otimes 3 = 6 \oplus \bar{3}$$

$$\langle \mathrm{TrL(R)TrL(0)} \rangle = \frac{1}{2} N(N+1) \cdot \exp(-N_t V_6(R)) + \frac{1}{2} N(N-1) \cdot \exp(-N_t V_{3^*}(R))$$

- Color-symmetric sextet（repulsive）

$$\mathrm{e}^{-V_6(R)} = \frac{3}{4} \langle \mathrm{TrL(R)TrL(0)} \rangle + \frac{3}{4} \langle \mathrm{TrL(R)L(0)} \rangle \qquad C = +\frac{1}{3}$$

- Color-antisymmetric anti-triplet（attractive）

$$\mathrm{e}^{-V_{3^*}(R)} = \frac{3}{2} \langle \mathrm{TrL(R)TrL(0)} \rangle - \frac{3}{2} \langle \mathrm{TrL(R)L(0)} \rangle \qquad C = -\frac{2}{3}$$

Partial-length Polyakov line correlator

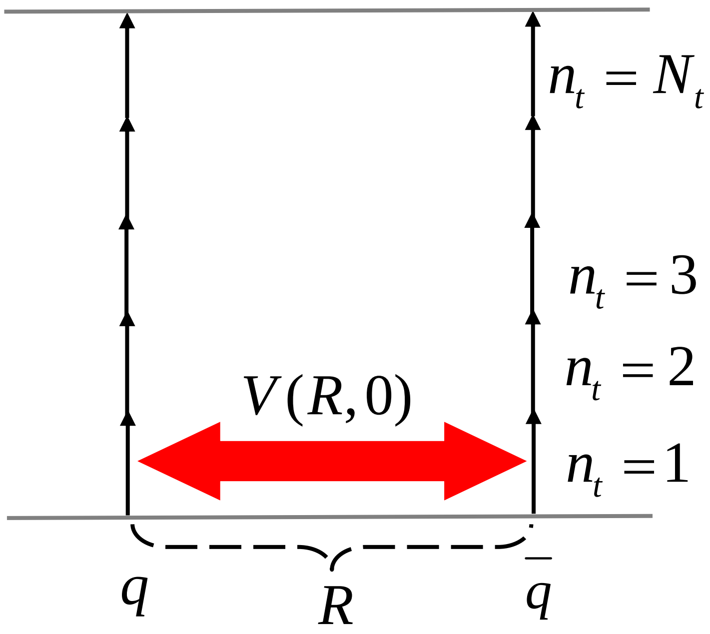
Correlators with partial length to the temporal direction. (Greensite, Olejnik, PRD67,094503(2003),PRD69,074506(2004).)

$$G(R,T) = \frac{1}{3}\left\langle Tr\left[L(R,T)L^{\dagger}(0,T)\right]\right\rangle,\ \mathrm{R}=\left|\vec{\mathrm{x}}\right|$$

$$V(R,T) = \log\frac{G(R,T)}{G(R,T+1)}$$

$$V(R,0) = -\log\left[G(R,1)\right]$$

Assume V(R,0) is instantaneous potential.



We can expect that instantaneous potential defined by link-link correlator has clear signal.

Related refs. on color-dep. force on the lattice (1)

1. Quark liberation at high temperature: A Monte Carlo study of SU(2) gauge theory, McLerran and Svetitsky, PRD 24, 450 (1981).
2. Non-Abelian Debye screening: The color-averaged potential Nadkarni, PRD 33, 3738 (1986).
3. Non-Abelian Debye screening. II. The singlet potential Nadkarni, PRD34,3904(1986).
4. Polyakov loop correlations in Landau gauge and the heavy quark potential. Attig, Karsch and Petersson, Satz and Wolff, PLB209,65(1988)
5. Heavy quark potentials in quenched QCD at high temperature Kaczmarek, et. al, PRD62, 034021 (2000)
6. Long-distance behavior of $q¥barq$ color dependent potentials at finite temperature, Nakamura and Saito, PTP 111 (2004) 733-743.
7. Heavy qq interaction at finite temperature, Nakamura and Saito, PTP 112 (2004) 183-188.

QGP physics at finite temperature

Related refs. on color-dep. force on the lattice (2)

1. QCD color interactions between two quarks.
   Nakamura and Saito, PLB621(2005)171

Only one in the hadron phase at zero temperature

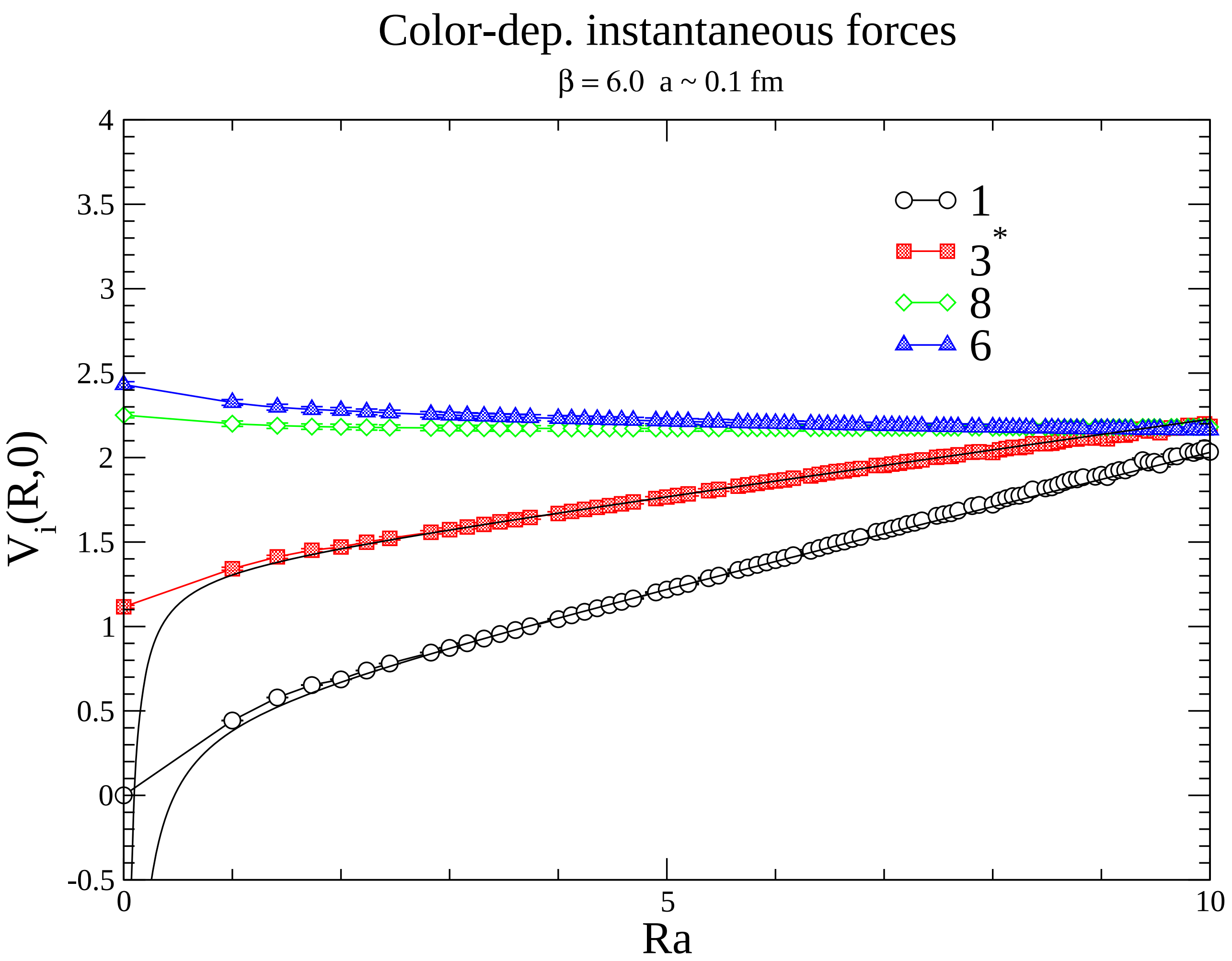
- Lattice calculations of the Polyakov line correlators become very noisy in the hadron phase.
- We may need some new device to deal with color-dependent pot.

Numerical results

Simulation parameters

- Wilson plaquette gauge action (simplest one) and quenched calculation.
- Hyper-cubic lattice with N=12,18,24,32.
- Gauge configuration number is 300 ~ 700.
- Gauge couplings, beta = 5.9-6.0, lattice cutoff a ~ 0.1 fm.
- Coulomb gauge fixing (and temporal gauge fixing) by numerical iterative method, a la Mandulor-Oglive.
- Precision of gauge fixing is up to $O(10^6)$.
- We use only the Polyakov line expectation values on the real-axis relating Z(3) symmetry.
- Computer facilities are SX-5 and SX-8 supercomputers at RCNP.

Color-dependent instantaneous forces



$C_1 = -\frac{4}{3}$, singlet

$C_{3^*} = -\frac{2}{3}$, anti-triplet

$C_8 = \frac{1}{6}$, octet

$C_6 = \frac{1}{3}$, sextet

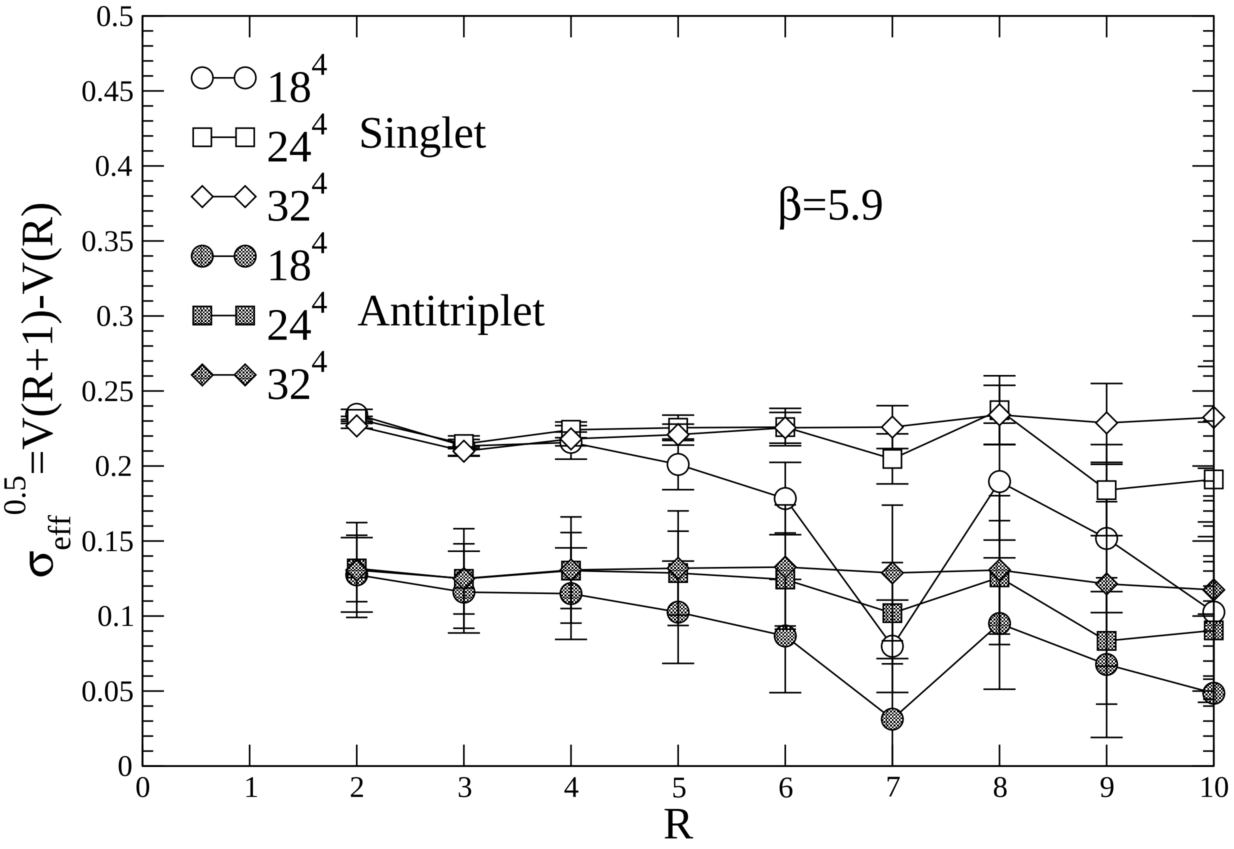
$$V(R) = c_0 + KR - \frac{A}{R}$$

$$C_1/C_{3^*} = (-4/3)/(-2/3) = 2$$

$$\sqrt{\sigma_1}a \sim 0.170 - 0.175$$

$$\sqrt{\sigma_{3^*}}a \sim 0.092 - 0.097$$

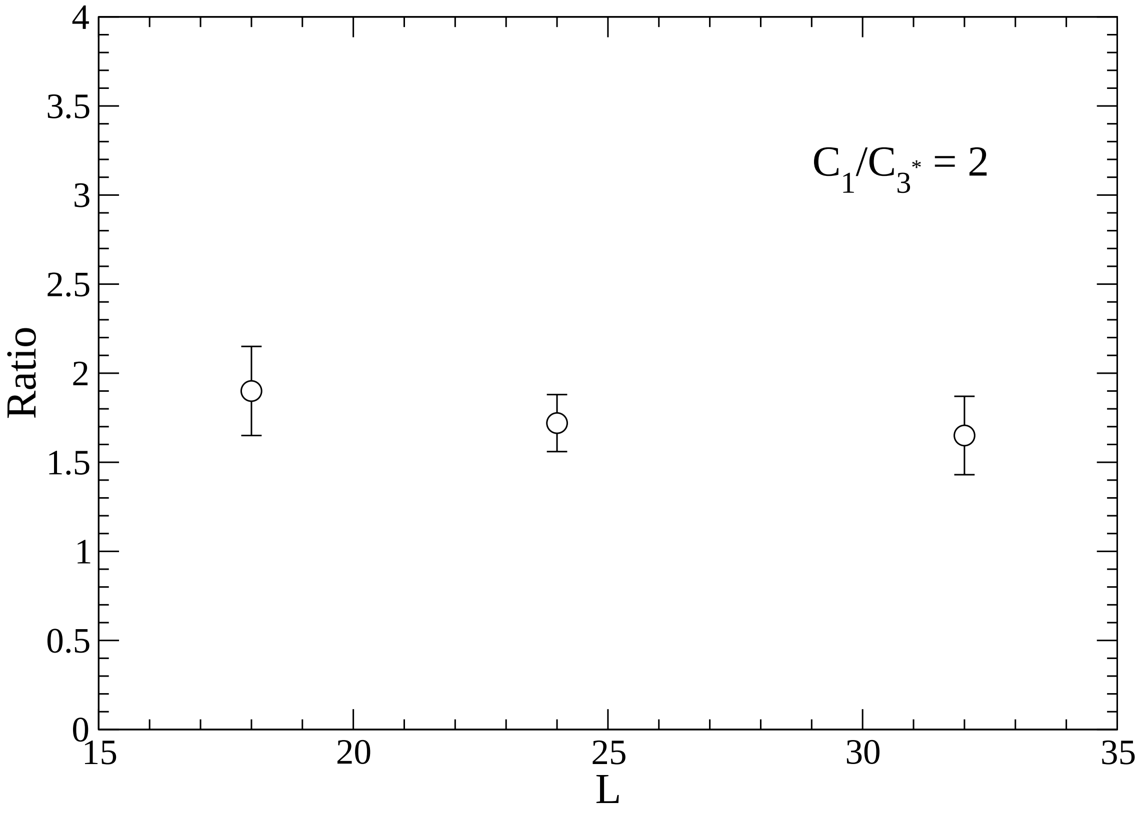
Effective string tensions



$$V(R) = c_0 + KR - \frac{A}{R}$$

$C_1/C_{3^*} = (-4/3)/(-2/3) = 2$

Ratios of the color forces between singlet and anti-triplet channels



$$V(R) = c_0 + KR - \frac{A}{R}$$

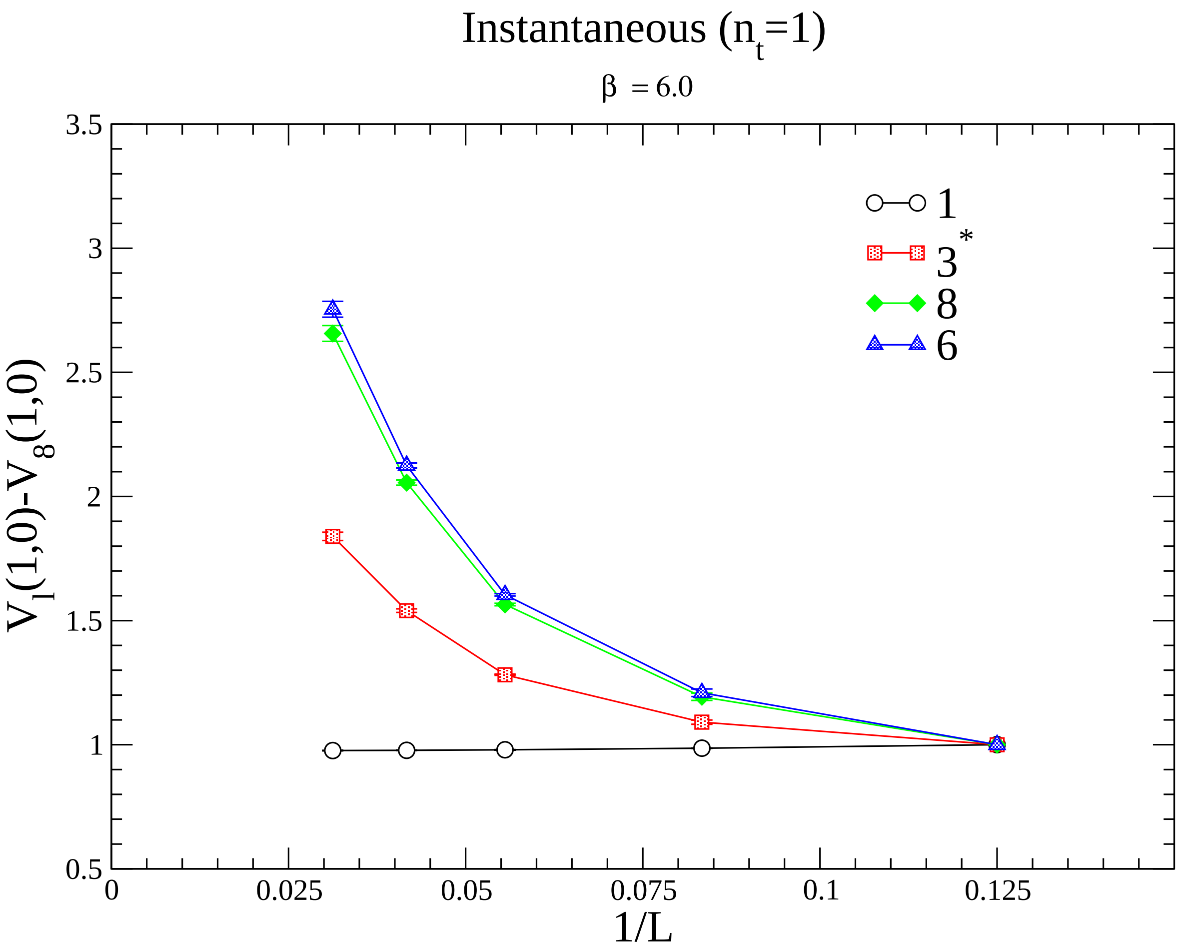
$$C_1/C_{3^*} = (-4/3)/(-2/3) = 2$$

Divergence of color flux in color non-singlet

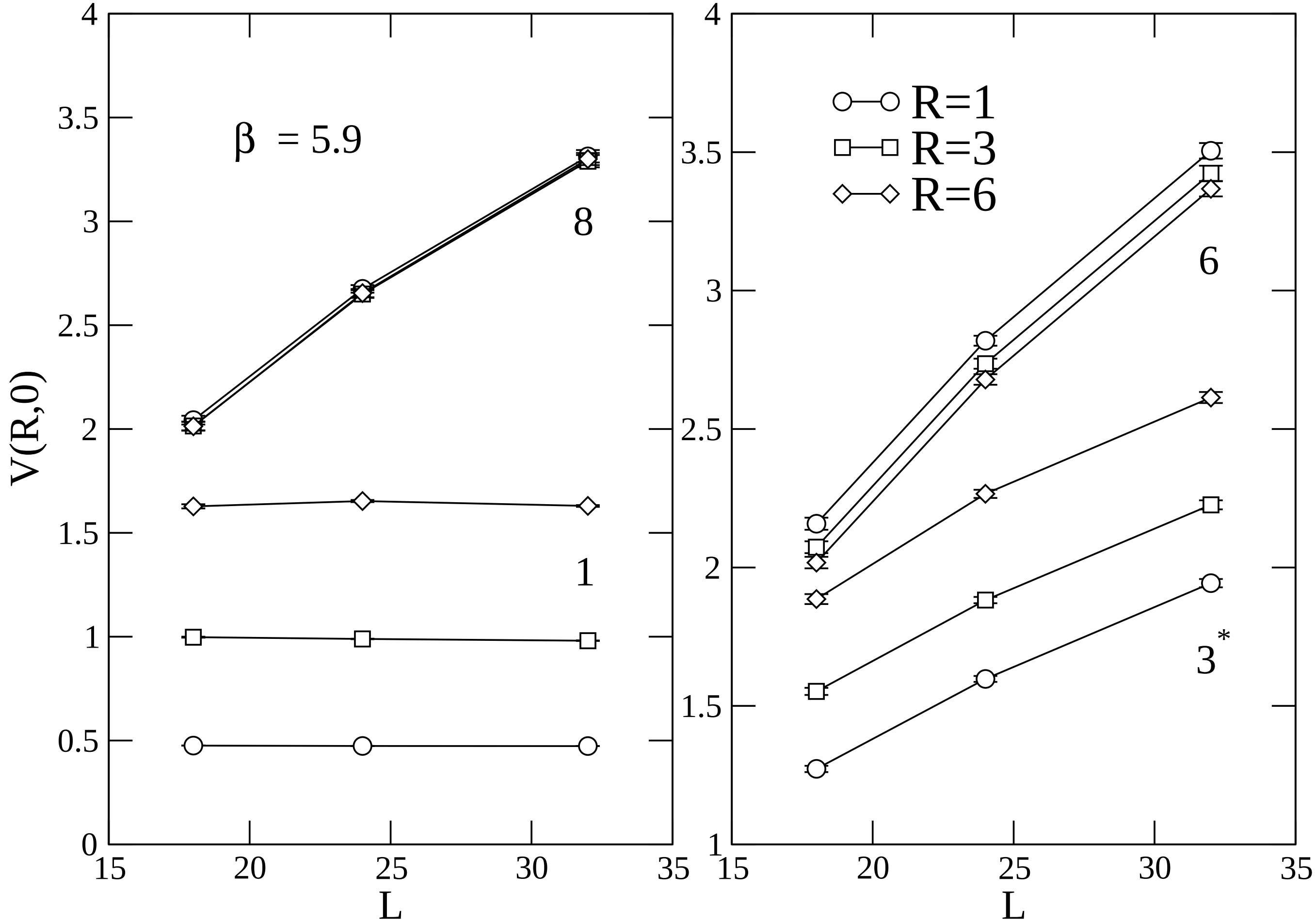
- Color-singlet color-flux should be closed or shrunk to a string. But, color non-singlet channels may not so.
- On the lattices, we may expect that there is a volume dependence in color non-singlet channels.

Volume effect（divergence of color flux）

- Singlet channel has little volume dependence.
- But, color non-singlet channels have large volume dependence; namely the color-flux of those diverges.



Volume effect（divergence of color flux）



Divergence part of color-Coulomb instantaneous potential

- Hamiltonian in the Coulomb gauge

$$H = \frac{1}{2}\int d^3x\left(E_i^2 + B_i^2\right) + \frac{1}{2}\int d^3x d^3y\left(\rho(x)D(x,y)\rho(y)\right)$$

- Color-Coulomb instantaneous potential

$$V_{inst}(r) = \langle D \rangle = \left\langle \frac{1}{M}\left(-\partial_i^2\right)\frac{1}{M} \right\rangle$$

- Color charge density

$$\rho_a \sim T_1^a \delta(x - x_0) + T_2^a \delta(x - y_0)$$

Divergence part of color-Coulomb instantaneous potential

- Instantaneous potential depending on the distance R

$$V_c(\vec{R}) = T_1^a T_2^b \int \frac{d\vec{p}}{(2\pi)^3} \frac{d^2(p) f(p)}{p^2} \exp(i\vec{p}\vec{R})$$

$d(p)$: Ghost form factor ~1/$\sqrt{\mathrm{p}}$

$f(p)$: Laplacian form factor ~ 1/p

$$= T_1^a T_2^b \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{p^4} \exp(i\vec{p}\vec{R})$$

$$= T_1^a T_2^b \frac{2\pi}{iR} \int_{-\infty}^{\infty} \frac{1}{p^4} \exp(i\vec{p}\vec{R})$$

$$= T_1^a T_2^b \frac{2\pi}{iR} \int_{-\infty}^{\infty} \frac{1}{p^3} \left(1 + ipR + \frac{1}{2}(ipR)^2 + \frac{1}{3}(ipR)^3 + \cdots \right)$$

$$\longrightarrow \quad V_c^{IS} = 4\pi (T_1^a T_2^b) \int_0^{\infty} dp \frac{1}{p^2}$$

Divergence part of color-Coulomb instantaneous potential

- Instantaneous potential not depending on the distance R

$$\Sigma_c = (T_i^a)^2 \int \frac{d\vec{p}}{(2\pi)^3} \frac{d^2(p) f(p)}{p^2}$$

$$= (T_i^a)^2 \int \frac{d\vec{p}}{(2\pi)^3} \frac{1}{p^4}$$

$$\longrightarrow \Sigma_c^{IS} = 4\pi (T_i^a)^2 \int_0^\infty dp \frac{1}{p^2}$$

Divergence part of color-Coulomb instantaneous potential

- Divergence parts

$$V_c^{IS} = 4\pi (T_1^a T_2^b) \int_0^\infty dp \frac{1}{p^2} \qquad \Sigma_c^{IS} = 4\pi (T_i^a)^2 \int_0^\infty dp \frac{1}{p^2}$$

$$(T_1^a T_2^b) + (T_i^a)^2 = (-4/3) + (4/3) = 0 \text{ for color-singlet}$$

$$(T_1^a T_2^b) + (T_i^a)^2 = (1/6) + (4/3) = 9/6 = 3/2 \text{ for color-octet}$$

$$(T_1^a T_2^b) + (T_i^a)^2 = (-2/3) + (4/3) = 2/3 \text{ for color-antitriplet}$$

$$(T_1^a T_2^b) + (T_i^a)^2 = (1/3) + (4/3) = 5/3 \text{ for color-antitriplet}$$

$$4:9:10 \text{ for } 3^*, 8 \text{ and } 6$$

Summary

1. We have tried to study the long-distance behavior of the color-dependent two quark potentials in the lattice QCD simulations.
2. Color-singlet and color anti-triplet (di-quark) channels yield the linearly rising potentials for large quark separations.
3. The instantaneous potentials seem to depend on the color (Casimir) factors qualitatively.
4. Color non-singlet flux diverges in the infinite volume limit.

Future works

- ✓We obtain clear signals of the potential including the vacuum polarizations ? Effect of vacuum polarization ?
- ✓Color-dependent three quark potentials ?
- ✓In particular, di-quark potential in 3 quark state ?
- ✓Maybe, we need any smearing techniques, in particular, for a calculation of full-length polyakov line correlator.