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Mass gap between spin multiplets of heavy mesons

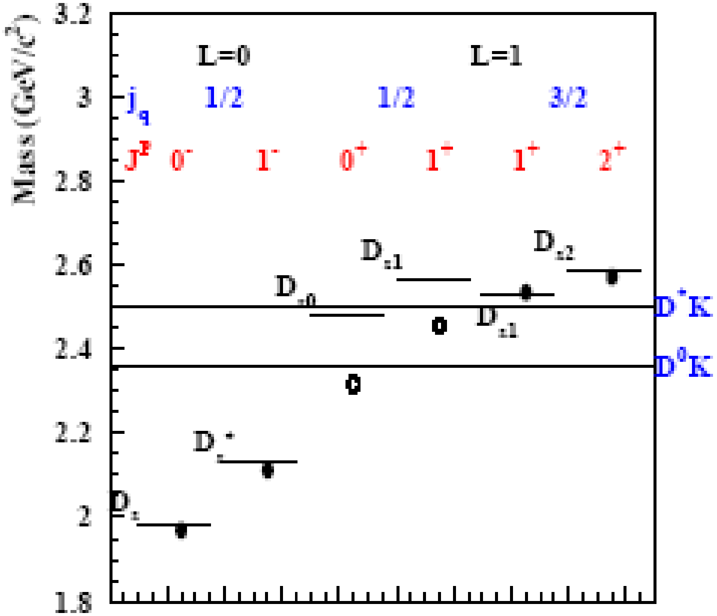
森井俊行（神戸大発達）、松木孝幸（東京家政大）、
須藤和敬（KEK)

1. **Introduction**
2. **Brief review of our Semirelativisitc Model**
3. **Mass Difference**
4. **Some Comments**

References:

Phys. Rev. D56, 5646 (1997)
Phys. Lett. B606, 329 (2005)
Prog. Theor. Phys. 117, 1077 (2007)

Ds mesons



(K. Abe, Talk at PENTAQUARK04, July 20-23, 2004.)

- ● **already observed**
- ○ **newly observed**
- — **prediction by conventional potential model** **(Godfrey et al., PRD43, 1679 (1991))**

There seem to be discrepancies between model predictions and observed values for L=1 (0^+ and 1^+) states.

People claim something odd must happen for these states.

D mesons

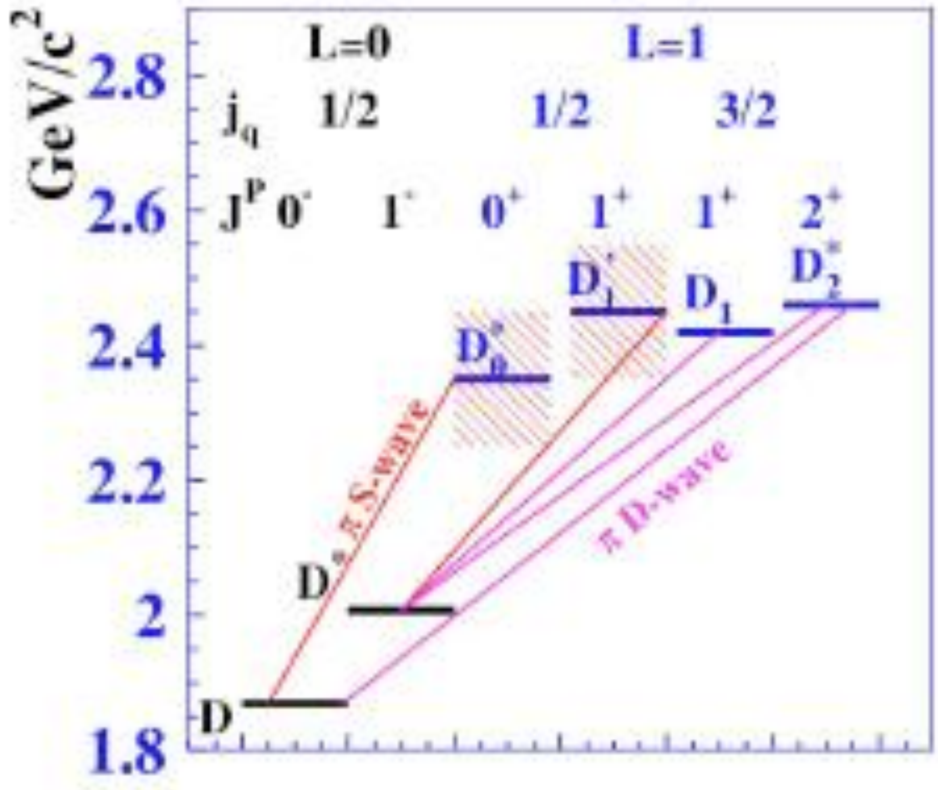


FIG. 1. Spectroscopy of *D*-meson excitations. The lines show possible single pion transitions.

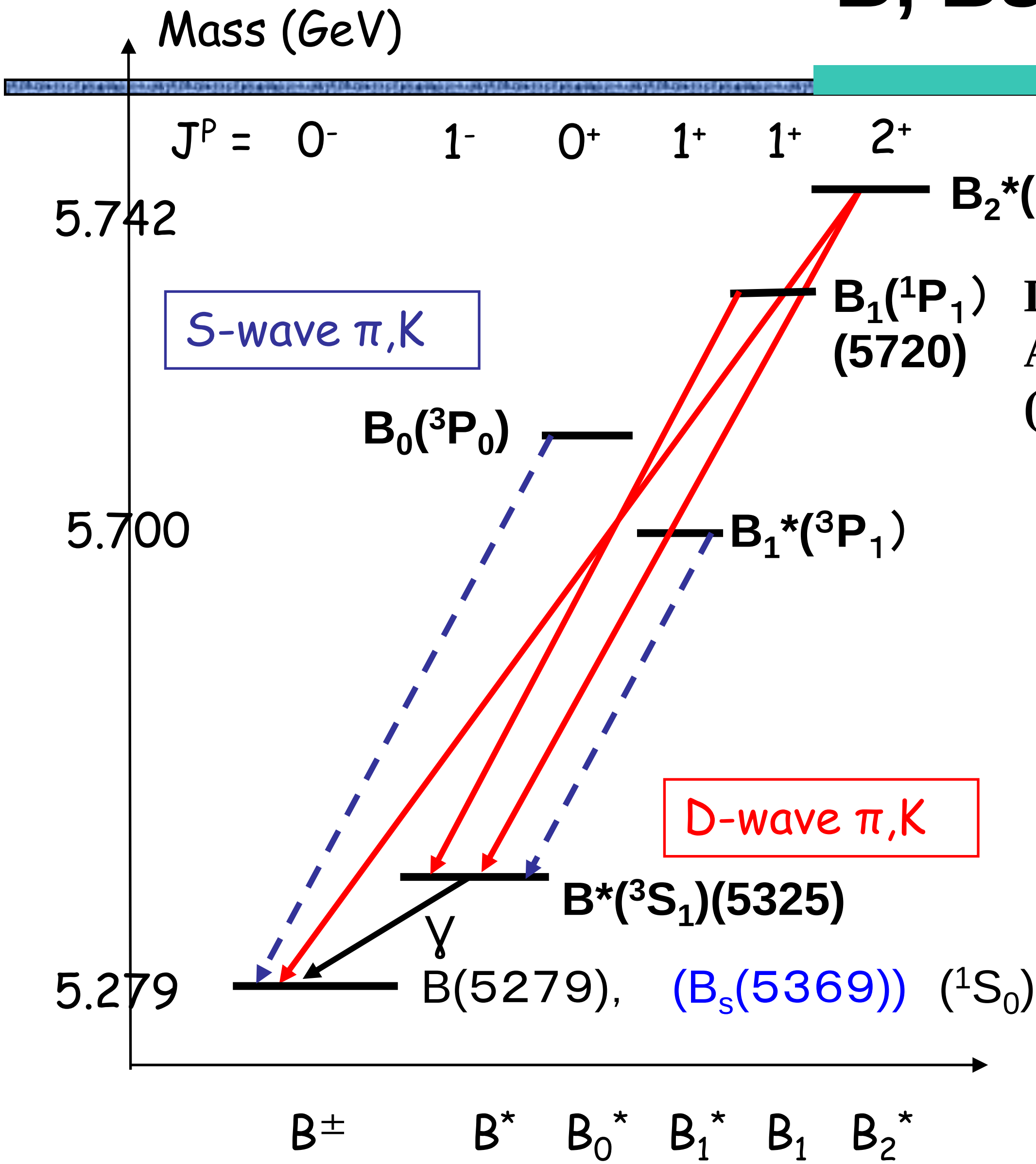
Belle collab. P. R. D69, 112002 (2004)

P state （L=1)

- Orbital angular momentum L=1
- Spin : 1/2 ×1/2 = 0 + 1

Orbital		spin		Total J	$^{2S+1}L_J$	ccbar	csbar	cubar
1	×	0	=	1	$^1P_1$ (1^+)		$D_{S1}(2535)$	$D_1(2420)$
1	×	1	=	0	$^3P_0$ (0^+)	$\chi_{c0}(3416)$	$D_{S0}(2317)$	$D_0(2308)$
			=	1	$^3P_1$ (1^+)	$\chi_{c1}(3511)$	$D_{S1}(2460)$	$D_1(2427)$
			=	2	$^3P_2$ (2^+)	$\chi_{c2}(3556)$	$D_{S2}(2572)$	$D_2(2460)$

B, Bs mesons



Denis Gelé, Talk at DIS2006, April 20-24, 2006 held at KEK. (D0 Collabration)

Notations are due to Eichten, Hill, Quigg, PRL 71, 4116 (1993) Valuse are from Pierro and Eichten, 64, 114004 (2001)

P state （L=1)

- Orbital angular momentum L=1
- Spin : 1/2 ×1/2 = 0 + 1

Orbital		spin		Total J	$^{2S+1}L_J$	bbbar	bsbar	bubar
1	×	0	=	1	$^1P_1$ (1^+)			$B_1(5720)$
1	×	1	=	0	$^3P_0$ (0^+)	$\chi_{b0}(9860)$		
			=	1	$^3P_1$ (1^+)	$\chi_{b1}(9893)$		
			=	2	$^3P_2$ (2^+)	$\chi_{b2}(9913)$	$B_{S2}(5839)$	$B_2(5745)$

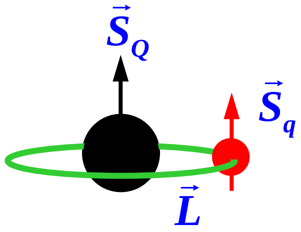
Many approach to explain spectra

- **Potential model**
  Godfrey, Isgur,,, PRD32,189(1985）;PRD43,1679(1991)
  Ebert, Galkin, Faustov, PRD57,5663(1998)
  Di Piero, Eichten, PRD64,114004(2001)
- **Effective Lagrangian approach**
  Bardeen, Hill, Eichten, PRD68, 054024(2003)
  Nowak, Rho, Zahed, PRD48, ４３７０（１９９３）
  Harada et. al., PRD70,074002(2004)
- **DK molecules**
  Barnes, Close, Lipkin, PRD68,054006(2003)
  Szczepaniak, PLB567,23(2003)
- **Four quark models**
  Terasaki, PRD68,011501(2003);Terasaki, McKeller, hep-ph/0501188
  Browder, Pakvasa, Petrov, PLB578,365(2004)
  Dmitrasinovic, PRL 94,162002(2005)
- **QCD sum rule**
  Hyashigaki, Terasaki, hep-ph/0411285
- **Lattice calculations**
  Bali, PRD68,071501(2003); UKQCD coll., PLB569,41,(2003)

Review: Swanson, Phys. Rep. 429, 243 (2006)

Our semi-relativistic model

- **Heavy-light System:** $Q\bar{q}$
  - → **hydrogen-like meson**



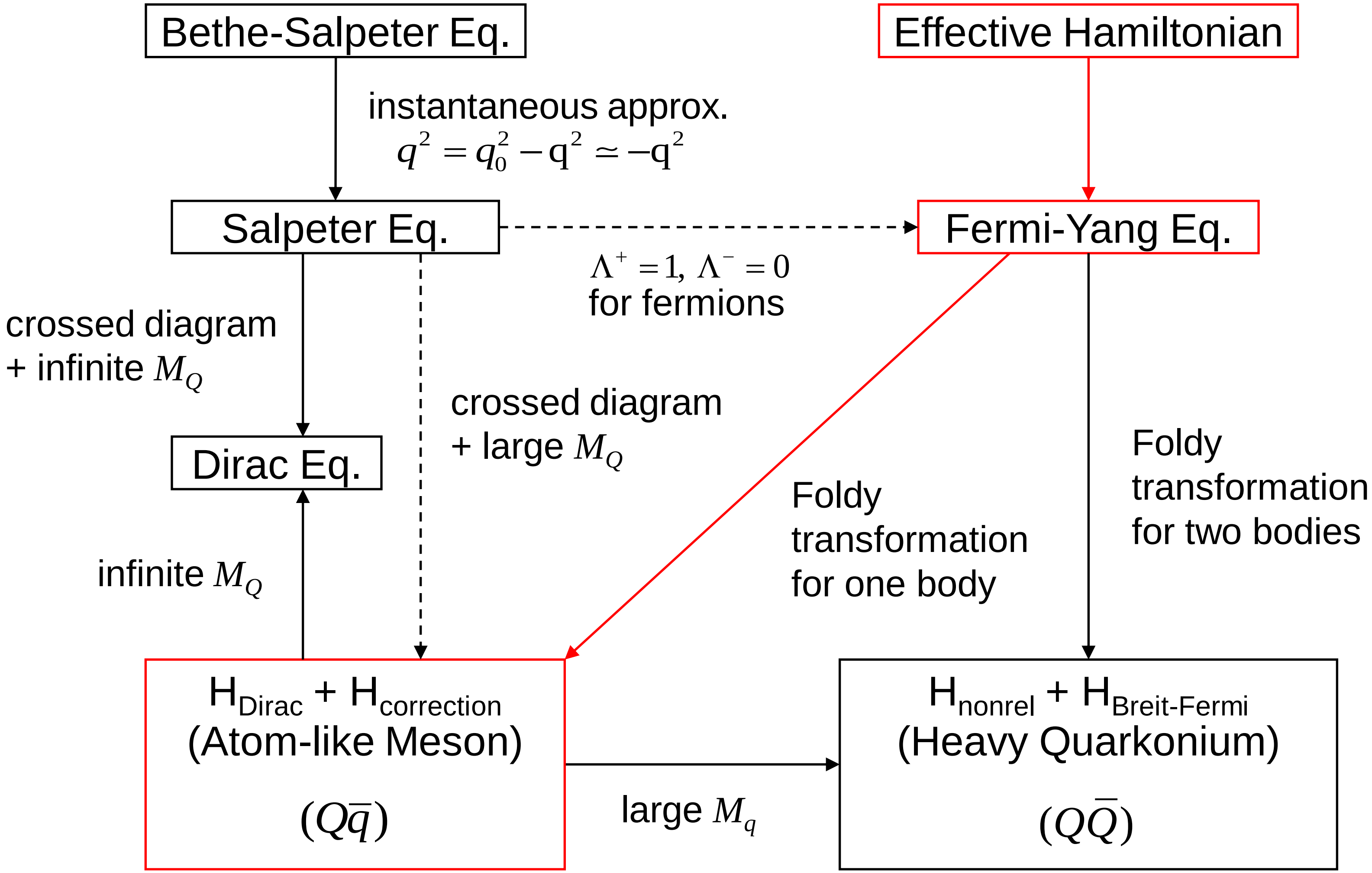
$$\vec{j}_q = \vec{L} + \vec{S}_q$$
$$\vec{J} = \vec{j}_q + \vec{S}_Q$$

–**We treat quarks as relativistically as possible and consistently take chiral and heavy quark symmetry into account, in the framework of the potential model.**

–**Hamiltonian and wave function are expanded in 1/m_Q .**

–**Include the contribution from both positive and negative components of the wave function.**

Reduction Schema for Two-body Bound State Eq.



Ref. Phys. Rev. D37, 159 (1988); Phys. Rep. 231, 201 (2003)

Effective Hamiltonian for $Q\bar{q}$ System

- **Start with an effective Hamiltonian for 2-body bound states**

$$H = (\vec{\alpha}_q \cdot \vec{p}_q + \beta_q m_q) + (\vec{\alpha}_Q \cdot \vec{p}_Q + \beta_Q m_Q) + \beta_q \beta_Q S$$
$$+ \left[1 - \tfrac{1}{2}\left\{\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})\right\}\right] V$$

$$S(r) = \frac{r}{a^2} + b, \quad V(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

Semi-relativistic approach to heavy meson

→ Foldy-Wouthuysen-Tani (FWT) transformation to the heavy quark ~1/m_Q expansion

- **Heavy Meson System**

$$H\psi_l = E^l \psi_l \qquad H = H_{\mathrm{FWT}} - m_Q = m_Q H_{-1} + H_0 + \frac{1}{m_Q} H_1 + \frac{1}{m_Q^2} H_2 + \cdots$$

$$E^l = E_0^l + \frac{1}{m_Q} E_1^l + \frac{1}{m_Q^2} E_2^l + \cdots$$

$$\psi_l = \psi_{l0} + \frac{1}{m_Q}\psi_{l1} + \frac{1}{m_Q^2}\psi_{l2} + \cdots$$

Effective Hamiltonian Expanded in $1/m_Q$

$$(H_{\text{FWT}} - m_Q) \otimes \psi_{\text{FWT}} = \tilde{E}\psi_{\text{FWT}}$$

$$H_{\text{FWT}} - m_Q = H_{-1} + H_0 + H_1 + H_2$$

$$H_{-1} = -(1+\beta_Q)m_Q$$

$$H_0 = \vec{\alpha}_q \cdot \vec{p} + \beta_q m_q - \beta_q \beta_Q S + \left\{1 + \tfrac{1}{2}[\vec{\alpha}_q \cdot \vec{\alpha}_Q + (\vec{\alpha}_q \cdot \vec{n})(\vec{\alpha}_Q \cdot \vec{n})]\right\} V$$

$$H_1 = -\frac{1}{2m_Q}\beta_Q \vec{p}^{\,2} + \frac{1}{m_Q}\beta_q \vec{\alpha}_Q \cdot \left(\vec{p} + \tfrac{1}{2}\vec{q}\right) S + \frac{1}{2m_Q}\vec{\gamma}_Q \cdot \vec{q} V$$

$$-\frac{1}{2m_Q}\left[\beta_Q\left(\vec{p} + \frac{1}{2}\vec{q}\right) + i\vec{q} \times \beta_Q \vec{\Sigma}_Q\right] \cdot \left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] V$$

$$H_2 = \frac{1}{2m_Q}\beta_q \beta_Q \left(\vec{p} + \frac{1}{2}\vec{q}\right)^2 S - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \beta_q \beta_Q \vec{\Sigma}_Q S - \frac{1}{8m_Q^2}\vec{q}^{\,2} V - \frac{i}{4m_Q^2}\vec{q} \times \vec{p} \cdot \vec{\Sigma}_Q V$$

$$-\frac{1}{8m_Q^2}\left\{(\vec{p} + \vec{q})(\vec{\alpha}_Q \cdot \vec{p}) + \vec{p}[\vec{\alpha}_Q \cdot (\vec{p} + \vec{q})] + i\vec{q} \times \vec{p}\gamma_Q^5\right\} \cdot \left[\vec{\alpha}_q + (\vec{\alpha}_q \cdot \vec{n})\vec{n}\right] V$$

$$\because \vec{p} = \vec{p}_q = -\vec{p}_Q, \quad \vec{p}\,' = \vec{p}\,'_q = -\vec{p}\,'_Q, \quad \vec{q} = \vec{p}\,' - \vec{p}$$

Eigenvalue equation without 1/m$_Q$ corrections

- **Eigenvalue equation:**

$$H_0 \otimes \psi_0^a = E_0^a \psi_0^a , \quad H_0 = \vec{\alpha}_q \cdot \vec{p}_q + \beta_q (m_q + \beta_Q S(r)) + V(r)$$

$$\psi_0^a = \begin{pmatrix} 0 & \Psi_{jm}^k(\vec{r}) \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{r} \begin{pmatrix} f_k(r) y_{jm}^k \\ i g_k(r) y_{jm}^{-k} \end{pmatrix} \end{pmatrix}$$

$$\vec{J} = \vec{j}_q + \tfrac{1}{2}\vec{\Sigma}_Q , \quad \vec{j}_q = \vec{L} + \tfrac{1}{2}\vec{\Sigma}_q$$

$$K = -\beta_q \left(\vec{\Sigma}_q \cdot \vec{L} + 1 \right), \quad K \Psi_{jm}^k = k \Psi_{jm}^k$$

j : total angular momentum of heavy meson
m: its z-component

k : quantum number of spinor operator K

- **The angular part is exactly solved.**

$$\begin{pmatrix} y_{jm}^{-(j+1)} \\ y_{jm}^{j} \end{pmatrix} = U \begin{pmatrix} Y_j^m \\ \vec{\sigma} \cdot \vec{Y}_{jm}^{(M)} \end{pmatrix}, \quad \begin{pmatrix} y_{jm}^{j+1} \\ y_{jm}^{-j} \end{pmatrix} = U \begin{pmatrix} \vec{\sigma} \cdot \vec{Y}_{jm}^{(L)} \\ \vec{\sigma} \cdot \vec{Y}_{jm}^{(E)} \end{pmatrix} \quad : U = \frac{1}{\sqrt{2j+1}} \begin{pmatrix} \sqrt{j+1} & \sqrt{j} \\ -\sqrt{j} & \sqrt{j+1} \end{pmatrix}$$

$$H_0 \otimes \psi_0^a = E_0^a \psi_0^a \quad \Rightarrow \quad \begin{pmatrix} m_q + S + V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - S + V \end{pmatrix} \Psi_k(r) = E_0^k \Psi_k(r) , \quad : \Psi_k(r) \equiv \begin{pmatrix} f_k(r) \\ g_k(r) \end{pmatrix}$$

depends on *k* alone

State Classification

- **Spinor operator K :** $K = -\beta_q \left(\vec{\Sigma}_q \cdot \vec{L} + 1 \right), \quad K \Psi^k_{jm} = k \Psi^k_{jm}$
  - **Relation between k and J**

$$k = \pm J \quad \text{or} \quad \pm (J+1) \longrightarrow \therefore J = |k| \quad \text{or} \quad |k| - 1 \quad (k \neq 0)$$

  - **The parity P of heavy meson**

$$P = \frac{k}{|k|} (-1)^{|k|+1}$$

Degeneracy is resolved due to $1/m_Q$ corrections

J^P	0^-	1^-	0^+	1^+	1^+	2^+
k	-1	-1	1	1	-2	-2
$j_q^{P_q}$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^+$
$^{2s+1}L_J$	$^1S_0$	$^3S_1$	$^3P_0$	$^3P_1, {}^1P_1$	$^1P_1, {}^3P_1$	$^3P_2$
Ψ^k_j	Ψ^{-1}_0	Ψ^{-1}_1	Ψ^1_0	Ψ^1_1	Ψ^{-2}_1	Ψ^{-2}_2

Chiral symmetry

- **The chiral symmetry is realized by setting** $m_q = S(r) = 0$**.**

$$H_0^{\text{chiral}} = \vec{\alpha}_q \cdot \vec{p} + V(r)$$

same as equation for a hydrogen atom with mass=0

Radial part equation becomes

$$\begin{pmatrix} V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & V \end{pmatrix} \Psi_k^{\text{chiral}}(r) = E_0^{k(\text{chiral})} \Psi_k^{\text{chiral}}(r)$$

This equation can be converted into the one with –k by unitary transformation.

$$\begin{pmatrix} V & -\partial_r - \frac{k}{r} \\ \partial_r - \frac{k}{r} & V \end{pmatrix} U\Psi_k^{\text{chiral}}(r) = E_0^{k(\text{chiral})} U\Psi_k^{\text{chiral}}(r) \qquad : U = \sigma_2$$

$$\therefore U\Psi_k^{\text{chiral}}(r) = \Psi_{-k}^{\text{chiral}}(r) \quad \text{and} \quad E_0^{k(\text{chiral})} = E_0^{-k(\text{chiral})}$$

Degeneracy between k and –k in a limit $m_q = S(r) \to 0$ **.**

Numerical Analysis

- **Input values to determine parameters**
  - **6 D meson masses**
  - **6 D_s meson masses**
  - **ground states of B and B_s mesons**

- **Most optimal values of parameters**

TABLE I: Optimal values of parameters.

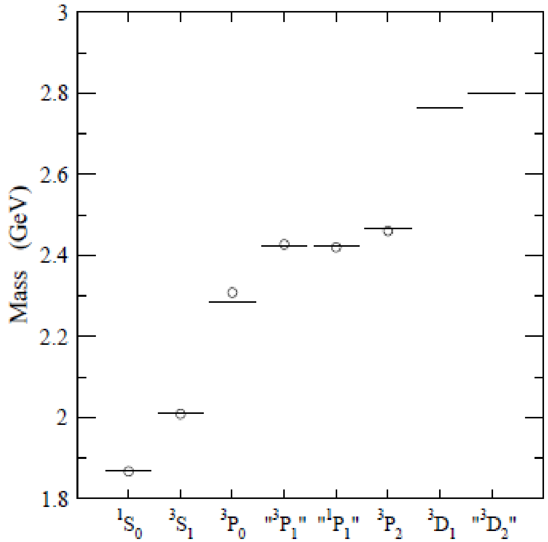
Parameters	α_s^c	α_s^b	a (GeV^{-1})	b (GeV)
	0.261±0.001	0.393±0.003	1.939±0.002	0.0749±0.0020
	$m_{u,d}$ (GeV)	m_s (GeV)	m_c (GeV)	m_b (GeV)
	0.0112±0.0019	0.0929±0.0021	1.032±0.005	4.639±0.005
	# of data	# of parameter	total χ^2/d.o.f	
	18	8	107.55	

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}, \quad S(r) = \frac{r}{a^2} + b \qquad \text{trial wave function: } \sim w_k(r)\left(\frac{r}{a}\right)^{\gamma} \exp\left[-(m_q+b)r - \frac{1}{2}\left(\frac{r}{a}\right)^2\right]$$

Using above parameters, other mass levels are calculated up to 1/m_Q^2.

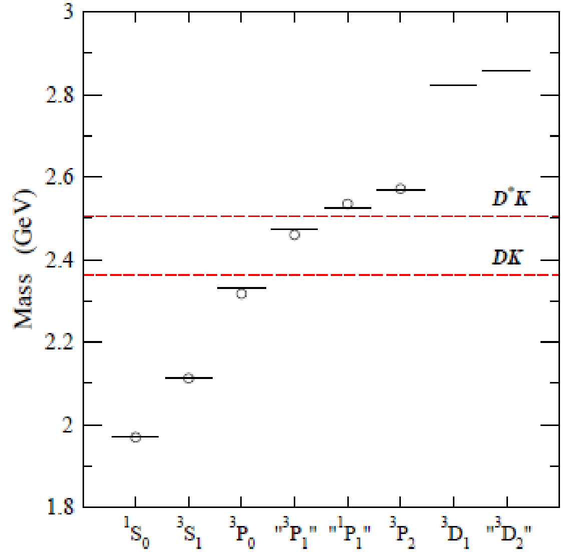
Light quark masses ane not input but outcome of χ^2 analysis.

D



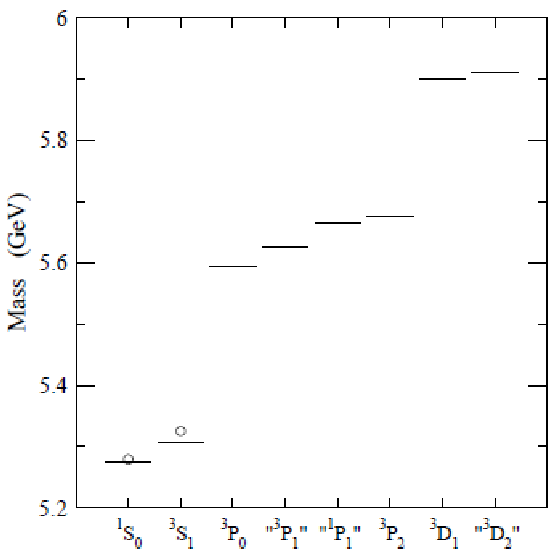
Mass (GeV)
3
2.8
2.6
2.4
2.2
2
1.8
$^1S_0$ $^3S_1$ $^3P_0$ "$^3P_1$" "$^1P_1$" $^3P_2$ $^3D_1$ "$^3D_2$"

D_s



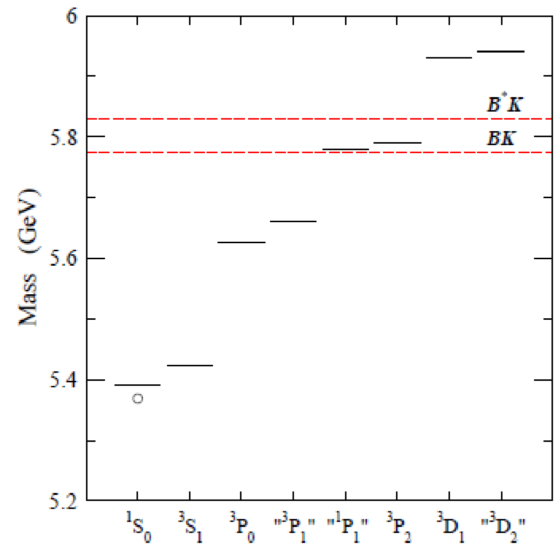
Mass (GeV)
3
2.8
2.6
2.4
2.2
2
1.8
D^*K
DK
$^1S_0$ $^3S_1$ $^3P_0$ "$^3P_1$" "$^1P_1$" $^3P_2$ $^3D_1$ "$^3D_2$"

B



Mass (GeV)
6
5.8
5.6
5.4
5.2
$^1S_0$ $^3S_1$ $^3P_0$ "$^3P_1$" "$^1P_1$" $^3P_2$ $^3D_1$ "$^3D_2$"

B_s



Mass (GeV)
6
5.8
5.6
5.4
5.2
B^*K
BK
$^1S_0$ $^3S_1$ $^3P_0$ "$^3P_1$" "$^1P_1$" $^3P_2$ $^3D_1$ "$^3D_2$"

Mass difference

Mass for the heavy meson X with the spin and parity, j^P, is expressed as

$$M_X(j^P) = m_Q + E_0^k(m_q) + O(1/m_Q),$$

where the quantum number k is related to the total angular momentum j and the parity P for a heavy meson as

$$j = |k| - 1 \quad \text{or} \quad |k|, \quad P = \frac{k}{|k|}(-1)^{|k|+1}, \quad E_0^k(m_q) = E_0(j^P, m_q).$$

In the chiral limit, m_q=0, S(r)=0, states with same |k| are degenerate.

Chiral symmetry is broken in two steps; first the degeneracy due to gluon fields is broken when S(r) is turned on but keeping vanishing light quark masses, → $\Delta M \approx 300\text{MeV} = \Lambda_{QCD}$

$$\Delta M = E_0(1^+,0) - E_0(1^-,0) = E_0(0^+,0) - E_0(0^-,0) = 295.088 \text{ MeV} \text{ for } D, \text{ and } D_s,$$

$$= 309.2 \text{ MeV} \text{ for } B, \text{ and } B_s,$$

then, turning on a light quark mass, which explicitly breaks chiral symmetry, mass of D becomes D_s as

$$\Delta M_0 = M_X(0^+) - M_X(0^-) = M_X(1^+) - M_X(1^-) = g_0\Lambda_{\text{QCD}} - g_1 m_q,$$

$$\Lambda_Q = 300 \text{ MeV}, \quad g_0 \approx 1, \quad g_1 \approx 1,$$

Structure of Mass Level

- We treat quarks as relativistically as possible and consistently take into account chiral & heavy quark symmetry.

Semi-relativistic approach

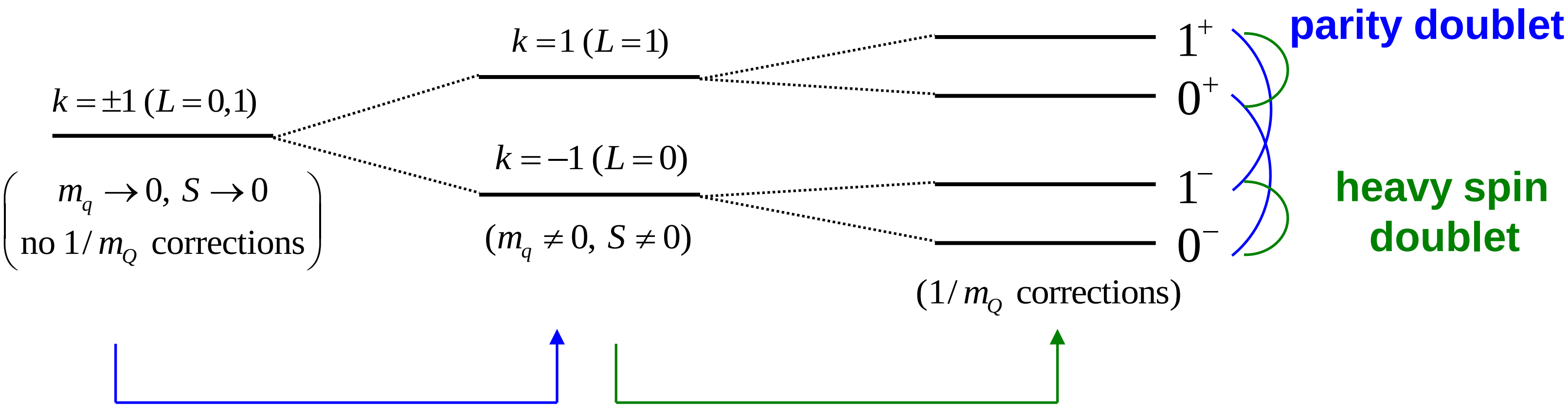
quark: relativistic, 4-spinor particle

heavy quark: (semi-)relativistic

+ $1/m_Q$ corrections: relativistic effects of heavy quark

  – Negative energy states both of q and Q are included.

- Structure of mass level



Chiral symmetry is broken.

Heavy quark symmetry is broken.

Degenerate mass gap $0^+(1^+)$-$0^-(1^-)$

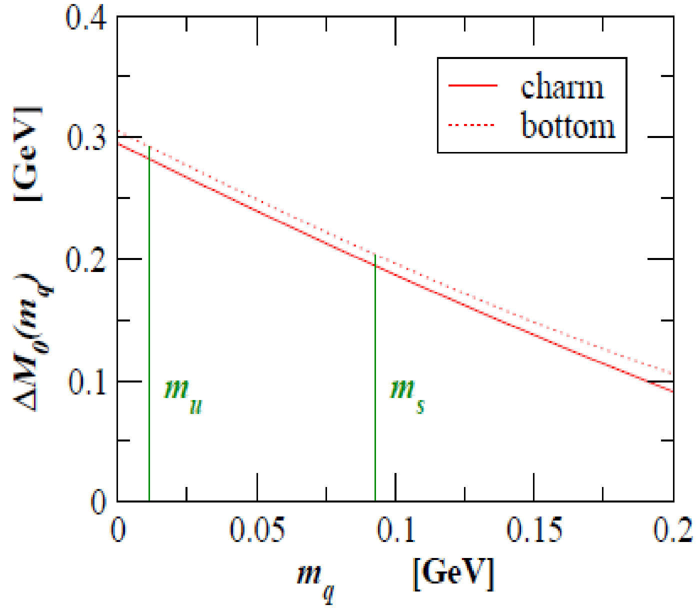


FIG. 1: Plot of the mass gap between two spin multiplets. Light quark mass dependence is given. The horizontal axis is light quark mass m_q and the vertical axis is the mass gap ΔM.

ΔM_0 **=295.1-1.080**m_q **for D, D_s mesons**
=305.2-1.089m_q **for B, B_s mesons**

Note that the mass gap depends negatively on the light quark mass.

m_q dependence of the mass gap:

$$\Delta M = M^1(0^+) - M^{-1}(0^-) = M^1(1^+) - M^{-1}(1^-)$$

$$= \int \frac{d^3x}{4\pi r^2}\left\{\Phi_1^\dagger(r)\begin{pmatrix} m_q+S+V & -\partial_r+\frac{1}{r} \\ \partial_r+\frac{1}{r} & -m_q-S+V\end{pmatrix}\Phi_1(r) - \Phi_{-1}^\dagger(r)\begin{pmatrix} m_q+S+V & -\partial_r-\frac{1}{r} \\ \partial_r-\frac{1}{r} & -m_q-S+V\end{pmatrix}\Phi_{-1}(r)\right\}$$

$$= \int dr\left[\Phi_1^\dagger(r)K_1\Phi_1(r) - \Phi_{-1}^\dagger(r)K_{-1}\Phi_{-1}(r)\right] + m_q\int dr\left[\Phi_1^\dagger(r)\beta\Phi_1(r) - \Phi_{-1}^\dagger(r)\beta\Phi_{-1}(r)\right].$$

where

$$\Phi_k(r) = \begin{pmatrix} u_k(r) \\ v_k(r)\end{pmatrix}, \quad K_k = \begin{pmatrix} S(r)+V(r) & -\partial_r+\frac{k}{r} \\ \partial_r+\frac{k}{r} & -S(r)+V(r)\end{pmatrix}$$

k=-1(+1) corresponds to L=0(L=1) states, then k=+1 is more relativisitc than k=-1, which means a lower component $v_1(r)$ is larger than $v_{-1}(r)$. Hence $(u_1)^2 - (v_1)^2 = \Phi_1^\dagger(r)\beta\Phi_1(r)$ becomes smaller than $\Phi_{-1}^\dagger(r)\beta\Phi_{-1}(r)$.

Thus, the coefficient of m_q becomes negative.

TABLE II: Degenerate masses of model calculations and their mass gap between $0^+(1^+)$ and $0^-(1^-)$ for $n=1$.

	$M_0(D)$	$M_0(D_s)$	$M_0(B)$	$M_0(B_s)$
$0^-/1^-$	1784	1900	5277	5394
$0^+/1^+$	2067	2095	5570	5598
$0^+(1^+)-0^-(1^-)$	283	195	293	204

$1/m_Q$ corrections:

We propose a mass gap formula including $1/m_Q$ corrections as

$$\Delta M = \Delta M_0 + \frac{c + d \cdot m_q}{m_Q}$$

with

$$c = 1.28 \times 10^5\ \mathrm{MeV}^2, \quad d = 4.26 \times 10^2\ \mathrm{MeV}$$

Where c and d are obtained from ΔM_0 and calculated mass gaps with $1/m_Q$ corrections for D and D_s mesons shown in Table below.

Applying this formula to B/B_s, we obtain :

$$B(0^+) - B(0^-) \approx B(1^+) - B(1^-) \approx 322, \quad B_s(0^+) - B_s(0^-) \approx B_s(1^+) - B_s(1^-) \approx 240 \quad (\mathrm{MeV}),$$

Which should be compared with calculation, 321 and 241MeV below.

TABLE III: Model calculations of the mass gap. Values in brackets are taken from the experiments. Units are MeV.

Mass gap ($n = 1$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	414 (441)	358 (348)	322	239
$1^+ - 1^-$	410 (419)	357 (348)	320	242

($n = 2$)	$\Delta M(D)$	$\Delta M(D_s)$	$\Delta M(B)$	$\Delta M(B_s)$
$0^+ - 0^-$	308	274	206	160
$1^+ - 1^-$	350	327	216	171

Mass gap with $1/m_Q$ corrections:

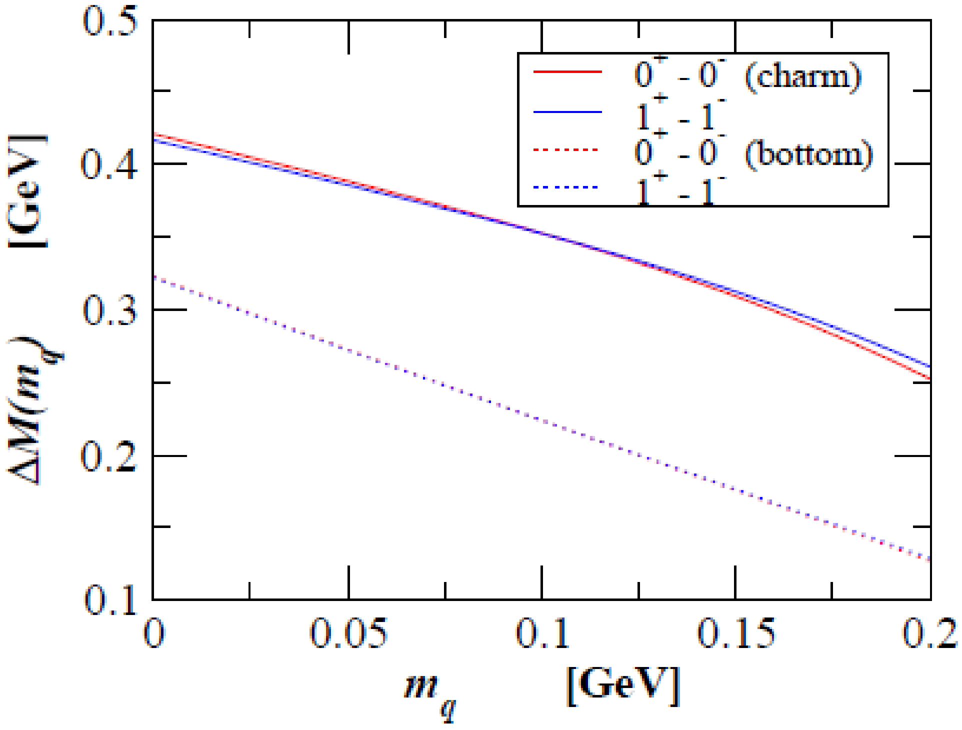


TABLE IV: D/D_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{\rm calc}(D)$	$M_{\rm obs}(D)$	$M_{\rm calc}(D_s)$	$M_{\rm obs}(D_s)$
$^1S_0(0^-)$	1869	1867	1967	1969
$^3S_1(1^-)$	2011	2008	2110	2112
$^3P_0(0^+)$	2283	2308	2325	2317
"$^3P_1$"(1^+)	2421	2427	2467	2460

TABLE V: B/B_s meson mass spectra for both the calculated and experimentally observed ones. Units are MeV.

$^{2s+1}L_J(J^P)$	$M_{\rm calc}(B)$	$M_{\rm obs}(B)$	$M_{\rm calc}(B_s)$	$M_{\rm obs}(B_s)$
$^1S_0(0^-)$	5270	5279	5378	5369
$^3S_1(1^-)$	5329	5325	5440	–
$^3P_0(0^+)$	5592	–	5617	–
"$^3P_1$"(1^+)	5649	–	5682	–

Effective Lagrangian approach

- **Bardeen et al. have proposed an effective Lagrangian approach which involves chiral symmetry in $m_q \to 0$, heavy quark symmetry in $m_Q \to \infty$.** (Bardeen et al., PRD68, 054024 (2003))

★ **(modified) Goldberger-Treiman relation in parity doublet**

$$\Delta M = g_\pi f_\pi, \qquad g_\pi:\ 0^+(1^+) \to 0^-(1^-)\pi \ \text{coupling constant}$$

$$f_\pi:\ \text{pion decay constant}$$

$$\text{cf.)}\quad g_{NN\pi} = m_N / f_\pi$$

⇨ $\Delta M(m_c) = \Delta M(m_b) = 349\ \text{MeV}$ (from experiments)

Only mass differences between two doublets (0^-,1^-) and (0^+,1^+) of D_s mesons are reproduced but no absolute values. Furthermore,

It is in fail for D mesons.

($m^*_{u,d}/m^*_s$)$\Delta M(m_c)$=255MeV ;

large SU(3) breaking effects?

Cf.

$D(0^+)-D(0^-)$=2308-1867=441MeV
$D(1^+)-D(1^-)$=2427-2008=419MeV

Recovery of SU(3) symmetry ?

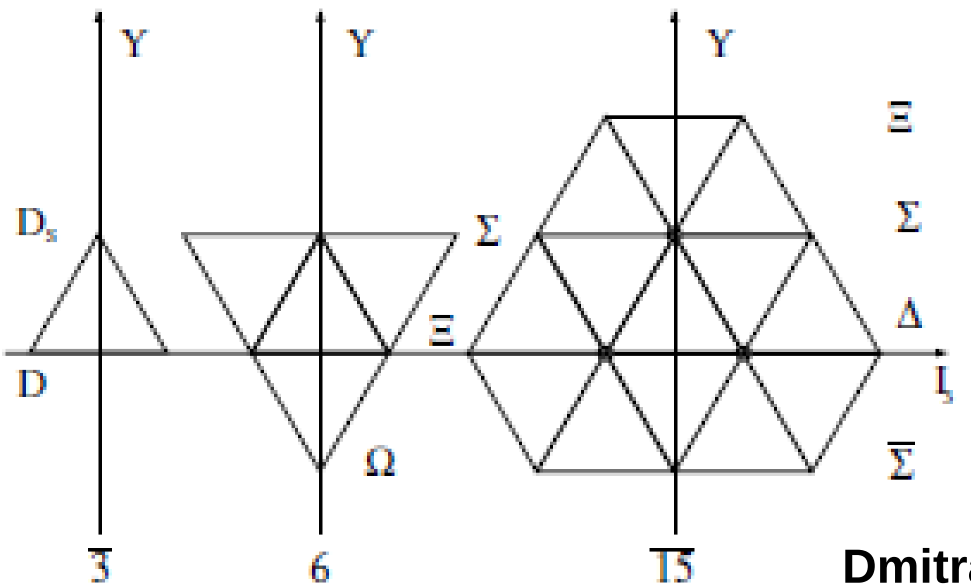
	S state (L=0)		P state (L=1)	
		S–P splitting (D): 414 (441)		
D :	1869 (0^-) (1867)		2283 (0^+) (2308)	
D–D_s splitting	102 (102)			42 (9)
D_s :	1967 (0^-) (1969)		2325 (0^+) (2317)	
		S–P splitting (D_s): 358 (348)		
		S–P splitting (D): 410 (419)		
D :	2011 (1^-) (2008)		2421 (1^+) (2427)	
D–Ds splitting	99 (104)			46 (33)
Ds :	2110 (1^-) (2112)		2467 (1^+) (2460)	
		S–P splitting (Ds): 357 (348)		

Tetra-quark model

$$|D^0 \subset \bar{3}_A\rangle = \frac{1}{2}|c(s(\bar{u}\,\bar{s} - \bar{s}\,\bar{u}) - d(\bar{d}\,\bar{u} - \bar{u}\,\bar{d}))\rangle,$$

$$|D^+ \subset \bar{3}_A\rangle = \frac{1}{2}|c(s(\bar{d}\,\bar{s} - \bar{s}\,\bar{d}) - u(\bar{d}\,\bar{u} - \bar{u}\,\bar{d}))\rangle,$$

$$|D_s^+ \subset \bar{3}_A\rangle = \frac{1}{2}|c(u(\bar{u}\,\bar{s} - \bar{s}\,\bar{u}) - d(\bar{d}\,\bar{s} - \bar{s}\,\bar{d}))\rangle,$$



Dmitrasinovic, PRL, 94,162002 (2005)

Summary and comment

1. Mass gap formula of two spin multiplets of heavy mesons is given by

$$\Delta M = \Delta M_0 + \frac{c + d \cdot m_q}{m_Q}$$ with $c = 1.28 \times 10^5\ \mathrm{MeV}^2, \quad d = 4.26 \times 10^2\ \mathrm{MeV}$

where

$$\Delta M_0 = M_X(0^+) - M_X(0^-) = M_X(1^+) - M_X(1^-) = g_0 \Lambda_{\mathrm{QCD}} - g_1 m_q,$$

$$\Lambda_Q = 300\ \mathrm{MeV}, \quad g_0 \approx 1, \quad g_1 \approx 1,$$

Mass gap decreases with light quark masses almost linearly.
This suggests that the physical ground of chiral symmetry breaking or mass generation of heavy mesons occurs differently from what peaple consider in effective chiral Lagrangian approach.

2. No SU(3) recovery for L=1 states; apparent SU(3) recovery is due to the fact that when D meson is lifted from $0^-/1^-$ states to $0^+/1^+$states, D_s meson is lifted about 50MeV smaller than that of D meson case.

3. As for physical ground of this mass gap formula, we need further investigation.