Dynamical $1/m_Q$ Corrections to Isgur-Wise Functions

T. Matsuki (Tokyo Kasei U.)
and
K. Seo (Gifu City Women's College)

Based on

- **Construction of Lorentz Invariant Amplitude from Rest Frame Amplitudes in HQET,** hep-ph/0703158.
- **New Heavy-Light Mesons Q qbar,** by T. M., T. Morii, adnd K. Sudoh, Prog. Theor. Phys. 117 (2007) 1077.
- **Spectroscopy of heavy mesons expanded in $1/m_Q$,** by T. M. and T. Morii, PRD56 (1997) 5646.

Motivations

- Discovery of narrow DsJ(2317) by BaBar and DsJ(2460) by CLEO in 2003.
- Subsequent discovery of broad $D_0^*(2308)$ and $D_1'(2427)$ by Belle.
- Recent discovery of narrow $B_1(5720)$ and $B_2^*(5745)$ by D0, which are considered to be 1^+ and 2^+ states of B, respectively.
- Also discovery of narrow $B_{s2}^*(5839)$ by D0, which are considered to be 2^+ states of B_s.
- More precise measurement of 0^- state of $B_c(6275)$ by CDF.
- Rich Spectra of Heavy-light Systems
- These can be either predicted or well reproduced by our model (T.M, T.M., and K.S.).
- *Needs to calculate branching widths to validate the model.*
- *(Isgur-Wise functions)*

Frame work1

- $\bar{B} \to D^{(*)} \ell \nu$ decay :
  - 6 form factors $\xi_i(\omega)$ (4 parameters $\rho_i(\omega)$)
- 4 frames
  - $\bar{B}$ rest
    - $D\left(u^c(t,\vec{x})c(t,\vec{y})\right)\left(t = x^0 = y^0\right)$
    - $D\left(u^c(t',\vec{x}')c(t',\vec{y}')\right)\left(t' = x'^0 = y'^0\right)$
  - Breit frame $\left(\beta = v_B = v_D\right)$
    - $\bar{B}\left(u^c(t,\vec{x})b(t,\vec{y})\right); \quad D\left(u^c(t,\vec{x})b(t,\vec{y})\right)$
    - $\bar{B}\left(u^c(t',\vec{x}')b(t',\vec{y}')\right); \quad D\left(u^c(t',\vec{x}')b(t',\vec{y}')\right)$
    - Lorentz boost in the rest frame

$$\begin{pmatrix} x'^0 \\ x'^3 \end{pmatrix} = \begin{pmatrix} \cosh\varphi & \sinh\varphi \\ \sinh\varphi & \cosh\varphi \end{pmatrix} \begin{pmatrix} x^0 \\ x^3 \end{pmatrix} \qquad \begin{pmatrix} y'^0 \\ y'^3 \end{pmatrix} = \begin{pmatrix} \cosh\varphi & \sinh\varphi \\ \sinh\varphi & \cosh\varphi \end{pmatrix} \begin{pmatrix} y^0 \\ y^3 \end{pmatrix}$$

Frame work2

- Matrix elements $\left\langle D^{(*)}; P'_D \{\ell'\} \middle| j(t,\vec{x}) \middle| \overline{B}; P_{\overline{B}} \{\ell\} \right\rangle$
  - $j(t,\vec{x}) = Q^{\dagger}{}_{D^{(*)}\alpha}(t,\vec{x}) O_{\alpha\beta} Q_{\overline{B}}(t,\vec{x})$
- $\overline{B}$ rest
  - $t = x^0 = y^0$ for D meson

$$
\begin{aligned}
&\left\langle D; P'_D \{\ell'\} \middle| j(0,\vec{0}) \middle| \overline{B}; P_{\overline{B}} \{\ell\} \right\rangle \\
&= \left\langle D; P'_D \{\ell'\} \middle| Q^{\dagger}{}_{D^{(*)}\alpha}(0,\vec{0}) O_{\alpha\beta} \int d^3x\, q^{c\dagger}(0,\vec{x}) q^c(0,\vec{x})\, Q_{\overline{B}\beta}(0,\vec{0}) \middle| \overline{B}; P_{\overline{B}} \{\ell\} \right\rangle \\
&\simeq O_{\alpha\beta} \int d^3x \left\langle D; P'_D \{\ell'\} \middle| Q^{\dagger}{}_{D^{(*)}\alpha}(0,\vec{0}) q^{c\dagger}{}_{\gamma}(0,\vec{x}) \middle| 0 \right\rangle \left\langle \middle| q^c(0,\vec{x}) Q_{\overline{B}\beta}(0,\vec{0}) \middle| \overline{B}; P_{\overline{B}} \{\ell\} \right\rangle \\
&= \int d^3x\, \mathrm{tr}\left[\psi_D^{\ell'\dagger}\left(\left(0,\vec{x}\right); P_D \right) \psi_B^{\ell}\left(\left(0,\vec{x}\right); \left(M_B, \vec{0}\right) \right) O^T \right] \\
&= \int d^3x\, G^*_{\gamma\delta} G^*_{\alpha\varepsilon} \psi^{\ell'*}_{D\delta\varepsilon}\left(\left(0, \vec{x}_\perp, \gamma^{-1} x^3\right); \left(M_D, \vec{0}\right) \right) O_{\alpha\beta} \psi^{\ell}_{B\gamma\beta}(\vec{x}) e^{-i\left(M_D - m_c\right)\gamma\beta x^3}
\end{aligned}
$$

0-th Order in 1/m$_Q$ (Isgur-Wise function)

- 0-th order

$$\xi_+(\omega)=\xi_V(\omega)=\xi_{A_1}(\omega)=\xi_{A_3}(\omega)=\xi(\omega)$$

$$\xi_-(\omega)=\xi_{A_1}(\omega)=0 \qquad \left(\omega=p_D\bullet p_B/(m_D m_B)=v_D\bullet v_B\right)$$

- Isgur-Wise function $\xi(\omega)$
  - $\bar{B}$ rest & $t=x^0=y^0$: $\xi(\omega)=\frac{1}{\omega}-\frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle+\frac{\beta^2}{4}$
  - $\bar{B}$ rest & $t'=x'^0=y'^0$: $\xi(\omega)=1-\frac{1}{6}\beta^2\omega\tilde{E}_D^2\left\langle r^2\right\rangle-\frac{\beta^2}{4}$
  - Breit & $t=x^0=y^0$: $\xi(\omega)=\gamma^{-2}-\frac{1}{6}\left(\frac{\beta}{2}\right)^2\gamma^2\left(\tilde{E}_D+\tilde{E}_B\right)^2\left\langle r^2\right\rangle$
  - Breit & $t'=x'^0=y'^0$: $\xi(\omega)=\gamma^{-2}-\frac{1}{6}\left(\frac{\beta}{2}\right)^2\gamma^{-2}\left(\tilde{E}_D+\tilde{E}_B\right)^2\left\langle r^2\right\rangle$
  - All These agree with

$$\xi(\omega)=1-\left(\frac{1}{2}+\frac{1}{3}\bar{\Lambda}^2\left\langle r^2\right\rangle\right)(\omega-1)$$

Numerical Results

- In the previous equation,

$$\bar{\Lambda} = \lim_{m_Q \to \infty} \left(M_X - m_Q \right) = \lim_{m_Q \to \infty} \tilde{E}_D = \lim_{m_Q \to \infty} \tilde{E}_B = 0.752 \text{ GeV}$$

$$\left\langle r^2 \right\rangle = 5.009 \text{ GeV}^{-2}$$

- $\xi(1) = 1$

$$\xi'(1) = -\frac{1}{2} + \frac{1}{3} \bar{\Lambda}^2 \left\langle r^2 \right\rangle = -1.44$$

- Which should be compared with
  - $\xi'(1) = -1.04$ by Ebert, Faustov, and Galkin (2006)
  - $\xi'(1) = -0.83$ by UKQCD lattice calculation (2002)
  - $\xi'(1) = -3/4$ by Yaouanc et al. as a lower bound

First Order in 1/m$_Q$

- Neubert & Rieckert decomposition

$$\xi_+(\omega)=\left[1+\left(\frac{1}{m_c}+\frac{1}{m_b}\right)\rho_1(\omega)\right]\xi(\omega), \qquad j_{\mu+}=c^\dagger\gamma^0\gamma_\mu b+c^\dagger\left(\frac{-i}{2m_c}\gamma^0\overleftarrow{D}^\nu\gamma_\nu\gamma_\mu+\frac{i}{2m_c}\gamma^0\gamma_\mu\vec{D}^\nu\gamma_\nu\right)b$$

$$\xi_+(\omega)=\left[-\frac{\bar{\Lambda}}{2}+\rho_4(\omega)\right]\left(\frac{1}{m_c}-\frac{1}{m_b}\right)\xi(\omega),$$

$$\xi_V(\omega)=\left[1+\frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)+\frac{1}{m_c}\rho_2(\omega)+\frac{1}{m_b}\big(\rho_1(\omega)-\rho_4(\omega)\big)\right]\xi(\omega),$$

$$\xi_{A_1}(\omega)=\left[1+\frac{\bar{\Lambda}}{2}\frac{\omega-1}{\omega+1}\left(\frac{1}{m_c}+\frac{1}{m_b}\right)+\frac{1}{m_c}\rho_2(\omega)+\frac{1}{m_b}\left(\rho_1(\omega)-\frac{\omega-1}{\omega+1}\rho_4(\omega)\right)\right]\xi(\omega),$$

$$\xi_{A_2}(\omega)=\frac{1}{\omega+1}\frac{1}{m_c}\left[-\bar{\Lambda}+(\omega+1)\rho_3(\omega)-\rho_4(\omega)\right]\xi(\omega),$$

$$\xi_{A_3}(\omega)=\left[1+\frac{\bar{\Lambda}}{2}\left(\frac{\omega-1}{\omega+1}\frac{1}{m_c}+\frac{1}{m_b}\right)+\frac{1}{m_c}\left(\rho_2(\omega)-\rho_3(\omega)-\frac{1}{\omega+1}\rho_4(\omega)\right)\right.$$
$$\left.+\frac{1}{m_b}\big(\rho_1(\omega)-\rho_4(\omega)\big)\right]\xi(\omega).$$

Some Formula for Bbar->D*+ell+nu

- Current : $j_\mu = c^\dagger \gamma^0 \gamma_\mu b + c^\dagger \left(\frac{-i}{2m_c} \gamma^0 \overleftarrow{D}^\nu \gamma_\nu \gamma_\mu + \frac{i}{2m_c} \gamma^0 \gamma_\mu \vec{D}^\nu \gamma_\nu \right) b$

- parmeters : $\rho_1(\omega) = \rho_2(\omega) = -\frac{1}{3} C^1 \overline{\Lambda} \left\langle r^2 \right\rangle (\omega - 1)$

  $\rho_3(\omega) = \rho_4(\omega) = 0$

- where

$$\tilde{E}_X = M_X - m_Q = C_X^0 + \frac{C_X^1}{m_Q} + \cdots$$

$$C^1 = 0.19022 \text{ MeV}^2$$

Some Formula2

- Differential decay rate for $\bar{B} \to D\ell\bar{\nu}$

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3}\left|V_{cb}\right|^2 M_D^3 \left(M_B + M_D\right)^2 \left(\omega^2 - 1\right)^{3/2} F_D(\omega)^2$$

- parmeters **:**

$$F_D(\omega)^2 = \xi_+(\omega) - \frac{1-r}{1+r}\xi_-(\omega), \quad F_D(\omega) = F_D(1)\left[1 - \rho_{\mathrm{D}}^2(\omega - 1) + \cdots\right], \quad r = \frac{M_D}{M_B}$$

- first-order numerical values in $1/m_Q$:
  - $F_D(1) = \xi_+(1) - \frac{1-r}{1+r}\xi_-(1) = 1.07, \quad F'_D(1) = -0.875 = -1.07 \times 0.818$
- which should be compared with
  - $F_D(1) = 0.966, \quad F'_D(1) = -0.966 \times 0.88 = -0.850$ by Ebert et al.
  - $\rho_{\mathrm{D}}^2 = 0.76 \pm 0.16 \pm 0.08$ by CLEO in 1999
  - $\rho_{\mathrm{D}}^2 = 0.69 \pm 0.14$ by Belle in 2002

CKM Matrix Element (Bbar->D+ell+nu)

- $F_D(1)|V_{cb}| = 0.0414 \pm 0.0064$ by us
- $F_D(1)|V_{cb}| = 0.040 \pm 0.002$ by Ebert et al.
- $F_D(1)|V_{cb}| = 0.039 \pm 0.002$ by CLEO in 1999
- $F_D(1)|V_{cb}| = 0.041 \pm 0.003$ by Belle in 2002
- These values lead to the CKM matrix element
- $|V_{cb}| = 0.0387 \pm 0.060$ by us
- $|V_{cb}| = 0.0415 \pm 0.0020$ by Ebert et al.
- $|V_{cb}| = 0.042 \pm 0.005 \pm 0.004$ by CLEO in 1999 with average $F_D(1)$
- $|V_{cb}| = 0.0419 \pm 0.0045 \pm 0.0053 \pm 0.0030$ by Belle in 2002 with $F_D(1)=0.98$ (Caprini)

Some Formula for Bbar->D*+ell+nu

- Differential decay rate for $\bar{B} \to D^{*} \ell \bar{\nu}$

$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} \left|V_{cb}\right|^2 M_{D^*}^3 \left(M_B - M_{D^*}\right)^2 \sqrt{\omega^2 - 1} \left(\omega + 1\right)^2 \left[1 + \frac{4\omega}{\omega + 1} \frac{1 - 2\omega r^* + r^{*2}}{\left(1 - r^*\right)^2}\right] F_{D^*}(\omega)^2$$

$$\left(1 - r^*\right)^2 \left[1 + \frac{4\omega}{\omega + 1} \frac{1 - 2\omega r^* + r^{*2}}{\left(1 - r^*\right)^2}\right] F_{D^*}(\omega)^2$$

$$= \left\{ 2\left(1 - 2\omega r^* + r^{*2}\right)\left(1 + \frac{\omega - 1}{\omega + 1} R_1(\omega)^2\right) + \left[\left(\omega - r^*\right) - \left(\omega - 1\right) R_1(\omega)\right]^2 \right\} \xi_{A_1}(\omega)^2$$

$$R_1(\omega) = \frac{\xi_V(\omega)}{\xi_{A_1}(\omega)}, \quad R_2(\omega) = \frac{\xi_{A_3}(\omega) + r^* \xi_{A_2}(\omega)}{\xi_{A_1}(\omega)}, \quad r^* = \frac{M_{D^*}}{M_B}$$

- which should be compared with the following results.

Numerical Results1

- Comparison : our=Ebert et al.

	our	CLEO [31]	BaBar [33]	Belle [32]	DELPHI [34]
R_1	1.39	1.18(30)(12)	1.396(60)(44)		
R_2	0.92	0.71(22)(7)	0.885(40)(26)		
$\rho^2_{h_{A_1}}$	0.86	$\begin{cases} 0.91(15)(6)^a \\ 1.61(9)(21)^b \end{cases}$	$\begin{cases} 0.79(6)^a \\ 1.145(59)(46)^b \end{cases}$	$\begin{cases} 0.81(12)^a \\ 1.35(17)^b \end{cases}$	$1.39(10)(33)^b$
$F_{D^*}\lvert V_{cb}\rvert$	0.0343(12)	$\begin{cases} 0.0360(20)^c \\ 0.0431(13)(18)^b \end{cases}$	$\begin{cases} 0.0328(5)^c \\ 0.0376(3)(16)^b \end{cases}$	$\begin{cases} 0.0315(12)^c \\ 0.0354(19)(18)^b \end{cases}$	$0.0377(11)(19)^b$

[a] linear fit of experimental data.
[b] fit using the form factor h_{A_1} parameterization [28].
[c] fit using form factor predictions of our model.

- first-order numerical values in $1/m_Q$:

$$F_D(1)=1,\quad F'_D(1)=\xi'(1)=-1.44=-\rho^2_{h_{A_1}}$$

$$R_1(1)=1.45,\quad R_1'(1)=-0.222,\quad R_2(1)=0.942,\quad R_2'(1)=0.0286$$

Numerical Results2

- Theoretical Comparison : our=Ebert et al.

Ref.	$R_1(1)$	$R_1'(1)$	$R_2(1)$	$R_2'(1)$
our	1.39	−0.23	0.92	0.12
[30]	1.25	−0.10	0.81	0.11
[27]	1.27	−0.12	0.80	0.11
[4]	1.35	−0.22	0.79	0.15
[35]	1.15		0.94	
[36]	1.01(2)		1.04(1)	

- first-order numerical values in $1/m_Q$:

$$F_D(1)=1,\quad F'_D(1)=\xi'(1)=-1.44=-\rho^2_{h_{A_1}}$$

$$R_1(1)=1.45,\quad R_1'(1)=-0.222,\quad R_2(1)=0.942,\quad R_2'(1)=0.0286$$

CKM Matrix Element (Bbar->D*+ell+nu)

- $F_{D^*}(1)|V_{cb}|=0.0380\pm0.0021$ by us
- $F_D(1)|V_{cb}|= 0.040 \pm 0.002$ by Ebert et al.
- $F_D(1)|V_{cb}|=0.039 \pm 0.002$ by CLEO in 1999
- $F_D(1)|V_{cb}|=0.041 \pm 0.003$ by Belle in 2002
- These values lead to the CKM matrix element
- $|V_{cb}|=0.0380\pm0.021$ by us
- $|V_{cb}|=0.0343 \pm 0.0012$ by Ebert et al.
- $|V_{cb}|=0.0360\pm0.020$ by CLEO in 1999

  $|V_{cb}|=0.0431\pm0.013\pm0.018$ with average $F_D(1)$
- $|V_{cb}|=0.0315 \pm 0.0012$ by Belle in 2002

  $|V_{cb}|=0.0354 \pm 0.0019 \pm 0.0018$ with $F_D(1)=0.98$ (Caprini)

Summary

- Isgur-Wise function in the four frames

$$\xi(\omega)=1-\left(\frac{1}{2}+\frac{1}{3}\bar{\Lambda}^2\langle r^2\rangle\right)(\omega-1),\quad \bar{\Lambda}=\lim_{m_Q\to\infty}\left(M_X-m_X\right)$$

$$\xi(1)=1,\quad \xi'(1)=-\left(\frac{1}{2}+\frac{1}{3}\bar{\Lambda}^2\langle r^2\rangle\right)$$

- First order in gives

$$\rho_1(\omega)=\rho_2(\omega)=-\frac{1}{3}C^1\bar{\Lambda}\langle r^2\rangle(\omega-1),\quad \rho_3(\omega)=\rho_4(\omega)=0,\quad M_X-m_X=\sum_i\frac{C^i}{\left(m_X\right)^i}$$

- Processes $\bar{B}\to D\ell\bar{\nu}$ and $\bar{B}\to D^*\ell\bar{\nu}$ give the CKM matrix element as

$$F_D(1)=1.07,\quad F'_D(1)=-0.875$$

$$F_D^*(1)=\xi(1)=1,\quad F_D^{*\prime}(1)=-1.09$$

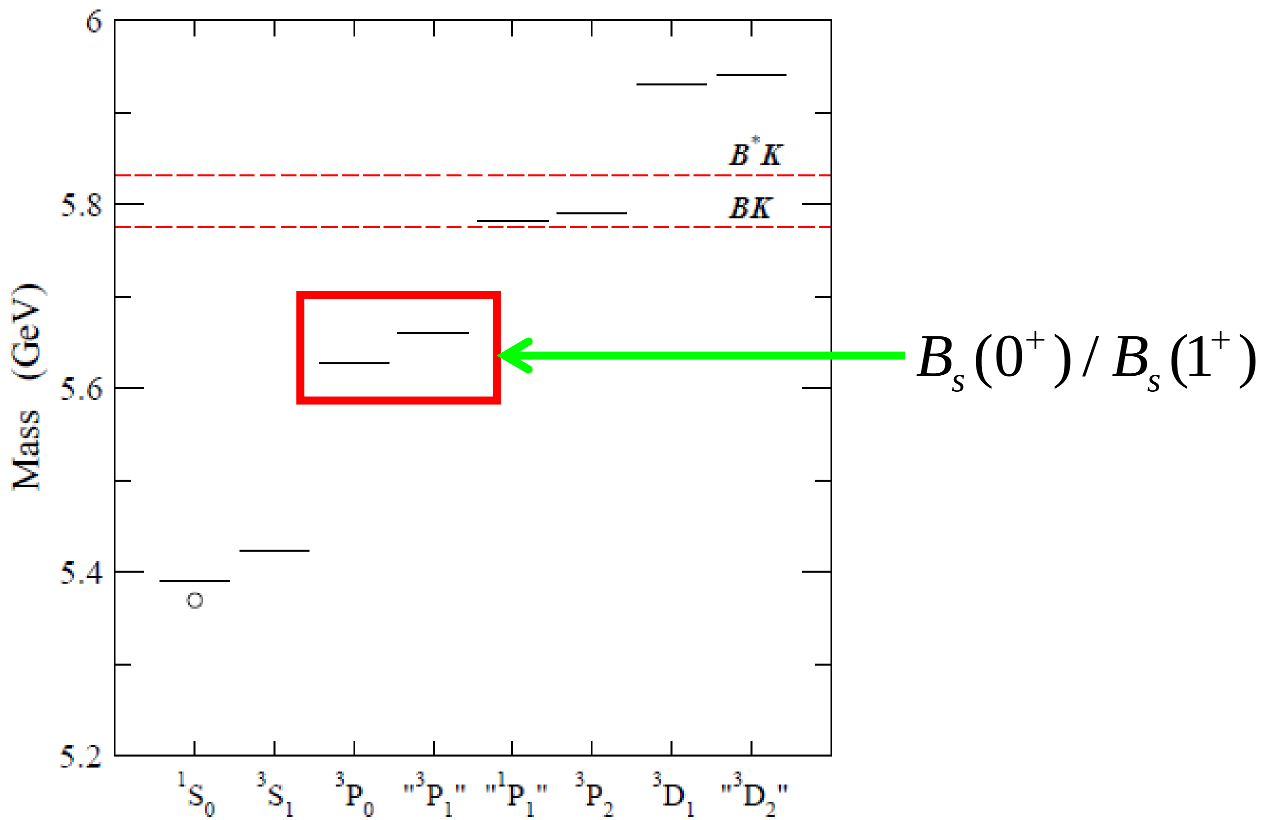
$$R_1(1)=1.45,\quad R'_1(1)=-0.222,\quad R_2(1)=0.942,\quad R'_2(1)=0.0286$$

$\left|V_{cb}\right|=0.0387\pm0.060$ for $\bar{B}\to D\ell\bar{\nu}$

$\left|V_{cb}\right|=0.0380\pm0.021$ for $\bar{B}\to D^*\ell\bar{\nu}$

Backup

- Find $B_s(0^+)/B_s(1^+)$ less than the BK/B^*K thresholds



Linear mq dependence of Mass Gap

- Underlying physics of the heavy-light system (ours)

  $$\Delta M = \Lambda_Q - m_q$$

- Effective chiral Lagrangian approach (Bardeen et al.)

  $$\Delta M = \Lambda_Q + m_q$$

- Which basic theory supports this criterion $\Delta M = \Lambda_Q - m_q$?

Other Tasks Left on Heavy Hadrons

- Can $Q\bar{Q}$ system be described by our idea?

- Construct formulation for heavy baryons.

- Compute decay amplitudes/widths.

- Supported works appearing:

- **De-Min Li1 , Bing Ma1 and Yun-Hu Liu1**
- **(1) Department of Physics, Zhengzhou University, Zhengzhou, Henan, 450052, P.R. China**

- **Received:** 11 November 2006 **Revised:** 8 February 2007 **Published online:** 3 May 2007

- **Abstract** In the framework of Regge phenomenology, the masses of the charmed states $c\bar{q}$ (q=u,d,s) lying on the 13S1-like trajectories are estimated. The overall agreement between our estimated masses and the recent predictions given using modified quark models by Matsuki et al., Lakhina et al. and Close et al. is good. The masses of the observed charmed states Ds0(2317), DsJ(2860) and DsJ(2690)/DsJ(2700) can reasonably be reproduced in the picture of these charmed states as simple quark–antiquark configurations. We therefore suggest that Ds0(2317) can be identified with the $c\bar{s}$(13P0) states. The possible assignments of the DsJ(2860) and DsJ(2690)/DsJ(2700) are discussed.